



**NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)**

(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)



DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE MATERIAL



EET 201 CIRCUITS AND NETWORKS

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to

imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- ◆ Established in: 2004
- ◆ Course offered: B.Tech Electrical and Electronics Engineering
- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

To excel in technical education and research in the field of Electrical & Electronics Engineering by imparting innovative engineering theories, concepts and practices to improve the production and utilization of power and energy for the betterment of the Nation.

DEPARTMENT MISSION

- To offer quality education in Electrical and Electronics Engineering and prepare the students for professional career and higher studies.
- To create research collaboration with industries for gaining knowledge about real-time problems.
- To prepare students with sound technical knowledge.
- To make students socially responsible.

PROGRAMME EDUCATIONAL OBJECTIVES

I. Graduates shall have the ability to work in multidisciplinary environment with good professional and commitment.

II. Graduates shall have the ability to solve the complex engineering problems by applying electrical, mechanical, electronics and computer knowledge and engage in lifelong learning in their profession.

III. Graduates shall have the ability to lead and contribute in a team with entrepreneur skills, professional, social and ethical responsibilities.

IV. Graduates shall have ability to acquire scientific and engineering fundamentals necessary for higher studies and research.

PROGRAM OUTCOMES

After the successful completion of the Course, B.Tech. Electrical and Electronics Engineering, **Graduates can able to acquire**

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to

provide valid conclusions.

5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Specific Outcomes (PSO)

1. Apply Science, Engineering, Mathematics through differential and Integral Calculus, Complex Variables to solve Electrical Engineering Problems.
2. Demonstrate proficiency in the use of software and hardware to be required to practice electrical engineering profession.
3. Able to apply the knowledge of Ethical and Management principles required to work in a team as well as to lead a team.

COURSE OUTCOME

After the completion of the course the student will be able to

CO 1	Apply circuit theorems to simplify and solve complex DC and AC electric networks.
CO 2	Analyse dynamic DC and AC circuits and develop the complete response to excitations.
CO 3	Solve dynamic circuits by applying transformation to s-domain.
CO 4	Analyse three-phase networks in Y and Δ configurations. Solve series /parallel resonant circuits.
CO 5	Develop the representation of two-port networks using network parameters and analyse.

CO VS PO'S AND PSO'S MAPPING

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PS01	PS02	PS03
CO 1	3	3	3	3	3	2	-	-	-	-	2	2	3	3	-
CO 2	3	3	3	3	3	2	-	-	-	-	2	2	3	3	-
CO 3	3	3	3	3	3	2	-	-	-	-	2	2	3	3	-
CO 4	3	3	3	3	3	2	-	-	-	-	2	2	3	3	-
CO 5	3	3	3	3	3	2	-	-	-	-	2	2	3	3	-

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

Module 1

Circuit theorems: DC and Sinusoidal steady state analysis of circuits with dependent and independent sources applying Superposition principle, Source transformation, Thevenin's, Norton's and Maximum Power Transfer theorems - Reciprocity theorem.

Module 2

Analysis of first and second order dynamic circuits: Formulation of dynamic equations of RL, RC and RLC series and parallel networks with dc excitation and initial conditions and complete solution using Laplace Transforms - Time constant - Complete solution of RL, RC and RLC circuits with sinusoidal excitation using Laplace Transforms – Damping ratio – Over damped, under damped, critically damped and undamped RLC networks.

Module 3

Transformed circuits in s-domain: Transform impedance/admittance of R, L and C - Mesh analysis and node analysis of transformed circuits in s-domain. Transfer Function representation – Poles and zeros.

Analysis of Coupled Circuits: – Dot polarity convention – Sinusoidal steady state analysis of coupled circuits - Linear Transformer as a coupled circuit - Analysis of coupled circuits in s-domain.

Module 4

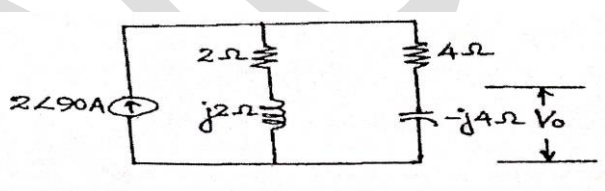
Three phase networks and resonance:Complex Power in sinusoidal steady state. Steady state analysis of three-phase three-wire and four-wire unbalanced Y circuits, Unbalanced Delta circuit, Neutral shift.

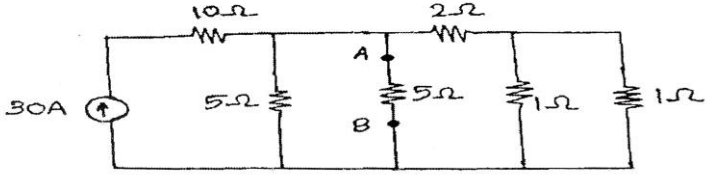
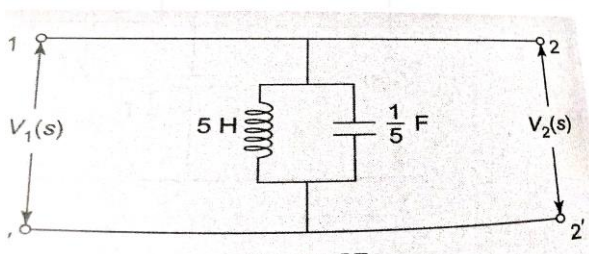
Resonance in Series and Parallel RLC circuits – Quality factor – Bandwidth – Impedance Vs Frequency, Admittance Vs Frequency, Phase angle Vs frequency for series resonant circuit.

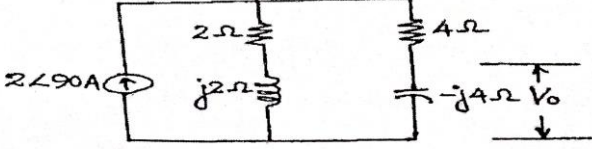
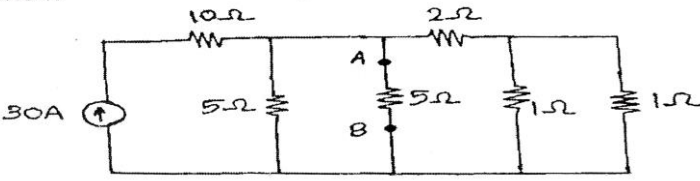
Module 5

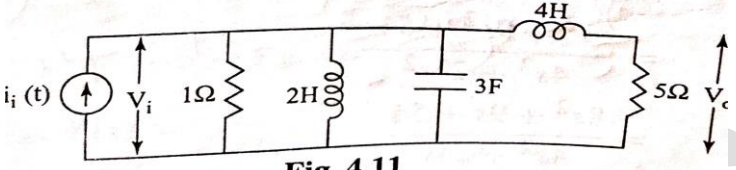
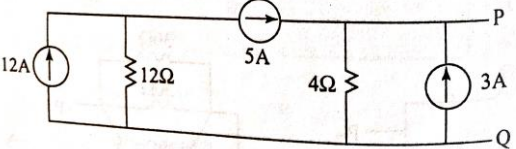
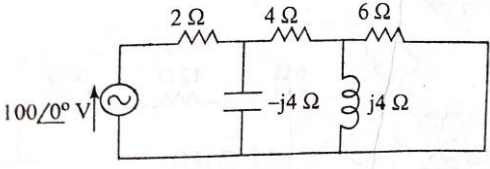
Two port networks: Driving point and transfer functions – Z, Y, h and T parameters - Conditions for symmetry & reciprocity – relationship between parameter sets – interconnections of two port networks (series, parallel and cascade) — T- π transformation.

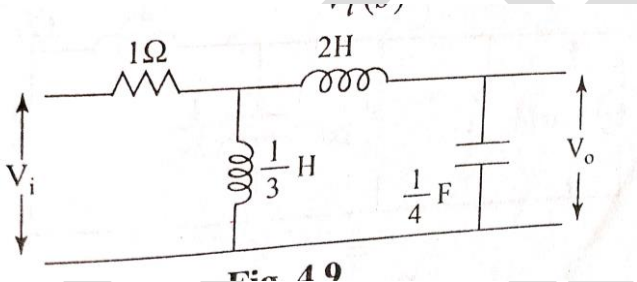
QUESTION BANK

Sl.No	Questions	KL/COL
1	Evaluate V_o in the following circuit and hence verify Reciprocity Theorem. 	K5/CO1
2	Using Norton Theorem, determine the current flowing through 5 ohm resistor in the circuit shown in figure.	K3/CO1

	below. 	
3	In a series RL circuit, the source voltage is $V(t) = 50 \sin 250t$ V, $R = 25$ ohm and $L = 0.005$ H. Determine the transient current when the switch is closed at $t = 0$. Assume zero initial conditions.	K3/CO2
4	In a series RLC circuit, the source voltage is $V(t) = 50$ V, $R = 35$ ohm and $L = 0.05$ H. Determine the transient current when the switch is closed at $t = 0$. Assume initial charge on the inductor as 3A and initial charge on the capacitor as 0.001C.	K3/CO2
5	For the given network function, draw the pole-zero plot and evaluate the stability of the system. $I(s) = 3s / ((s+1)(s+3))$	K5/CO3
6	Evaluate the transfer impedance of the given network . Calculate Voltage and Current Transfer Ratio. 	K5/CO3
7	In the following circuit, verify Superposition Theorem and evaluate current through 4ohm resistor. Voltage source in	K5/CO1

8	loop 2 is 6 angle 10 degree volts .  Using Thevenin's Theorem, evaluate the current flowing through 5 ohm resistor in the circuit shown in figure. below. 	K5/CO1
9	In a series RL circuit, the source voltage is $V(t) = 20 \sin 300t$ V, $R = 18$ ohm and $L = 0.05$ H. Determine the transient current when the switch is closed at $t = 0$. Assume zero initial conditions.	K3/CO2
10	In a series RLC circuit, the source voltage is $V(t) = 40$ V, $R = 25$ ohm and $L = 0.08$ H. Evaluate the transient current when the switch is closed at $t = 0$. Assume initial charge on the inductor as 2A and initial charge on the capacitor as 0.002C.	K5/CO2
11	For the given network function, draw the pole-zero plot and evaluate the stability of the system. $N(s) = (2s+1) / (4(s^2+5s+6))$. Calculate the number of poles and zeros at infinity.	K5/CO3

12	Evaluate i) $V_i(s) / I_i(s)$ ii) $V_o(s) / I_i(s)$ for the given network.  Fig. 4.11	K5/CO3
13	In the following circuit, verify Norton Theorem across the terminal PQ. 	K4/CO1
14	Using Thevenin's Theorem, evaluate the current flowing through 6 ohm resistor in the circuit shown in figure. 	K5/CO1
15	In a series RL circuit, the source voltage is $V(t) = 20 \sin 300t$ V, $R = 18$ ohm and $L = 0.05$ H. Determine the transient current when the switch is closed at $t = 0$. Assume zero initial conditions.	K3/CO2
16	In a series RLC circuit, the source voltage is $V(t) = 40$ V,	K5/CO2

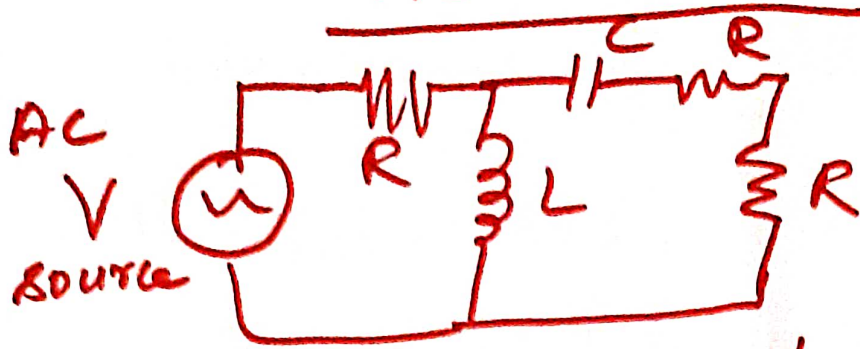
	R= 25 ohm and L =0.08H. Evaluate the transient current when the switch is closed at t=0. Assume initial charge on the inductor as 2A and initial charge on the capacitor as 0.002C.	
17	For the given network function, draw the pole-zero plot and evaluate the stability of the system. $N(s) = (s^2+s+2) / (s^3+3s^2+4s+2)$. Calculate the number of poles and zeros at infinity.	K5/CO3
18	Evaluate i) $V_i(s) / I_i(s)$ ii) $V_o(s) / I_i(s)$ for the given network. 	K5/CO3
19	A balanced three phase source of 400 V , 50 Hz supplies an unbalanced three phase load through an line has impedance of $0.1+j0.3\Omega$, $2+j5\Omega$ and $8+j6\Omega$. Determine line voltage , phase voltage, neutral - shift voltage, wire currents (line currents) , active power, reactive power, apparent power and power factor.	K6/CO4
20	A balanced three –phase system supplies an unbalanced delta – connected load made up of three resistors 200 ohm 400 ohm and 500 ohm and a coil having an inductance of	K6/CO4

	0.3H with negligible resistance. The line – to – line voltage is 100V and the supply frequency is 50Hz . Calculate the total power in the system and the total volt – ampere (reactive)	
21	Determine the Transmission parameters and Y parameters for the network. Check the condition for symmetry and reciprocity. $Z_{11} = 8 \text{ ohm}$, $Z_{12} = 12 \text{ ohm}$, $Z_{21} = 16 \text{ ohm}$, $Z_{22} = 18 \text{ ohm}$. Draw the equivalent circuits.	K6/CO5
22	Determine the Z parameters and h parameters for the network. Check the condition for symmetry and reciprocity. $A = 3$, $B = 160$, $C = 0.05$, $D = 3$. Draw the equivalent circuits.	K6/CO5

CONTENT BEYOND THE SYLLABUS

AC circuits

(1)



AC power supply

R, L, C

L - Inductor - Henry - H

C - Capacitor - Farad - F

Rectangular Form = $A + iB$
add, sub

Polar Form = $r \angle \theta$
multiply, divide

$$A + iB \Rightarrow r \angle \theta$$

$$r = \sqrt{A^2 + B^2}$$

$$\theta = \tan^{-1}(B/A)$$

$$r \angle \theta \Rightarrow A + iB$$

$$A = r \cos \theta$$

$$B = r \sin \theta$$

(2)

Impedance $Z = \text{Sum of } R, L, C$

Inductive Reactance

$$X_L = 2\pi f L$$

$$\pi = 3.14$$

$$f = \text{frequency} = 50 \text{ Hz}$$

$$L = \text{Inductor}$$

$$L = \frac{X_L}{2\pi f}$$

Capacitive Reactance

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi X_C}$$

(3)

$$X = \overset{A}{5} + j \overset{B}{5} = A + jB$$

find polar form $r \angle \theta$

$$r = \sqrt{A^2 + B^2} = \sqrt{5^2 + 5^2} = 7.071$$

$$\theta = \tan^{-1}(B/A) = \tan^{-1}(5/5) = 45^\circ$$

$$\text{Ans: } 7.071 \angle 45^\circ$$

$10 \angle 40^\circ$, find rectangular form

$$A = r \cos \theta = 10 \cos 40^\circ = 7.66$$

$$B = r \sin \theta = 10 \sin 40^\circ = 6.427$$

$$7.66 + j 6.427 \Rightarrow \text{rectangular form.}$$

$$C = 4 - j3, D = -8 + j10$$

$$C + D$$

$$= 4 - j3 - 8 + j10 = -4 + j7$$

$$C - D$$

$$= 4 - j3 - (-8 + j10)$$

$$= 4 - j3 + 8 - j10 = 12 - j13$$

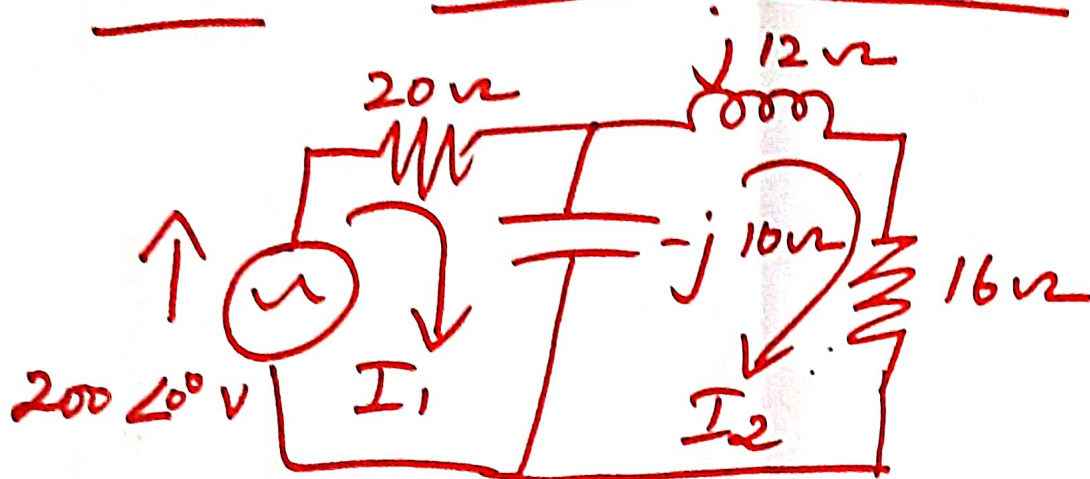
(11)

$$A = 10 \angle 40^\circ, B = 5 \angle 10^\circ$$

$$AB = 10 \times 5 \angle 40^\circ + 10^\circ = 50 \angle 50^\circ$$

$$\frac{A}{B} = \frac{10 \angle 40^\circ}{5 \angle 10^\circ} = 2 \angle 40^\circ - 10^\circ = 2 \angle 30^\circ$$

KVL - AC circuits



AC voltage source - AC circuit
R, L, C

Two closed loop = Two loop currents

= 2x2 matrix

$$\begin{bmatrix} \mathbf{Z} \\ \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{V} \\ \mathbf{V} \end{bmatrix}$$

same loop current direction

$$\begin{bmatrix} 20 - j10 & (-) - j10 \\ (-) - j10 & 16 - j10 + j12 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 200 \angle 0^\circ \\ 0 \end{bmatrix}$$

a_{11} a_{12} a_{21} a_{22}

$$\begin{vmatrix} 20-j10 & j10 \\ j10 & 16+j2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 200 \angle 0^\circ \\ 0 \end{vmatrix} \quad (5)$$

$$[(20-j10)(16+j2)] - [j10 \times j10]$$

$$(320 + j40 - j160 - j^2 20) - (j^2 100)$$

$$j^2 = -1, \quad -j^2 = 1$$

$$320 - j120 + 20 + 100 = 440 - j120 = \Delta.$$

Using Cramer's Rule

$$I_1 = \frac{\Delta I_1}{\Delta}$$

$$\Delta I_1 = \begin{vmatrix} 200 \angle 0^\circ & +j10 \\ 0 & 16+j2 \end{vmatrix}$$

$200 \angle 0^\circ$ = rectangular form.

$$A = r \cos \theta = 200$$

$$B = r \sin \theta = 0$$

$$= 200 + j0$$

$$\Delta I_1 = \begin{vmatrix} 200 + j0 & j10 \\ 0 & 16 + j2 \end{vmatrix}$$

$$(200)(16 + j2) - (0 \times j10)$$

$$3200 + j400 = \Delta I_1$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{3200 + j400}{440 - j120} = \cancel{3224.90}$$

Find Polar form for multiplication, and Division
 $3200 + j400$ $440 - j120$

$$r = \sqrt{A^2 + B^2}$$

$$= \sqrt{3200^2 + 400^2}$$

$$\theta = \tan^{-1} \left(\frac{B}{A} \right)$$

$$= \tan^{-1} \left(\frac{400}{3200} \right)$$

$$= 7.125^\circ$$

$$3224.90 \angle 7.125^\circ$$

polar form

$$r = \sqrt{440^2 + (-120^2)}$$

$$= 456.07$$

$$\theta = \tan^{-1} \left(\frac{-120}{440} \right)$$

$$= -15.255^\circ$$

$$456.07 \angle -15.255^\circ$$

polar form.

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{3200 + j400}{440 - j120} = \frac{3224.90 \angle 7.125^\circ}{456.07 \angle -15.255^\circ} \quad (7)$$

$$= \frac{3224.90}{456.07} \left\{ \angle 7.125^\circ - (-15.255^\circ) \right\}$$

$$= 7.071 \angle 7.125^\circ + 15.255^\circ$$

$$= 7.071 \angle 22.38^\circ \text{ A}$$

$$I_1 = A = r \cos \theta = 7.071 \cos 22.38^\circ = 6.538$$

$$B = r \sin \theta = 7.071 \sin 22.38^\circ = 0.2936$$

$$I_1 = 6.538 + j0.2936 \text{ A}$$

$$I_2 = \frac{\Delta I_2}{\Delta}$$

$$\Delta I_2 = \begin{vmatrix} 20 - j10 & 200 \angle 0^\circ \\ j10 & 0 \end{vmatrix}$$

$$200 \angle 0^\circ = 200 + j0$$

$$\Delta I_2 = \begin{vmatrix} 20 - j10 & 200 + j0 \\ j10 & 0 \end{vmatrix}$$

$$= [(20 - j10)0] - [j10(200 + j0)]$$

$$= -(j2000 + j^2 0)$$

$$= -j2000$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{-j2000}{440 - j120} = \frac{2000 \angle 0^\circ}{456.07 \angle -15.255^\circ}$$

Polar form for multiplication and Division

$$-j2000$$

$$A = 0$$

$$B = -2000$$

$$r = \sqrt{A^2 + B^2}$$

$$= \sqrt{2000^2}$$

$$= 2000$$

$$\tan^{-1}(B/A)$$

$$\tan^{-1}\left(\frac{-2000}{0}\right) = 0$$

$$= 2000 \angle 0^\circ$$

$$I_2 = \frac{2000}{456.07} \angle 0 - (-15.255^\circ)$$

$$= 4.39 \angle 15.255^\circ \text{ A}$$

$$r \cos \theta = 4.39 \times \cos 15.255^\circ$$

$$= 4.24$$

$$r \sin \theta = 4.39 \times \sin 15.255^\circ$$

$$= 1.16$$

$$I_2 = 4.24 + j1.16 \text{ A}$$

$$\text{Load Power} = P_L = I_L^2 R_L = 7.438 \text{ W}$$

Maximum Power transfer theorem: (AC circuits)

Maximum power is transferred to the load when the load impedance is equal to the complex conjugate of the impedance of the network looking back at it from the load terminals, all sources being replaced by their respective internal impedances.

Proof: Consider the circuit shown in the figure,

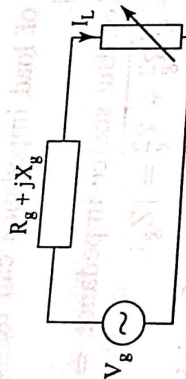


Fig. 2.187

$$\text{Load current, } I_L = \frac{V_g}{\sqrt{(R_g + R_L)^2 + (X_g + X)^2}}$$

$$\text{Power delivered to the load} = I_L^2 R_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + (X_g + X)^2}$$

For this power to be maximum, $\frac{\partial P_L}{\partial R_L} = 0$ and $\frac{\partial P_L}{\partial X} = 0$

$$\frac{\partial P_L}{\partial X} = \frac{[(R_g + R_L)^2 + (X_g + X)^2] \cdot 0 - V_g^2 R_L [2(X_g + X)]}{[(R_g + R_L)^2 + (X_g + X)^2]^2} = 0$$

$$\Rightarrow X_g + X = 0 \Rightarrow \boxed{X = -X_g}$$

$$\therefore \frac{\partial P_L}{\partial R_L} = \left\{ \frac{[(R_g + R_L)^2 + (X_g + X)^2] - R_L \cdot 2(R_g + R_L)}{[R_g + R_L)^2 + (X_g + X)^2]^2} \right\} V_g^2 = 0.$$

$$\Rightarrow (R_g + R_L)^2 + (X_g + X)^2 = 2R_L(R_g + R_L)$$

$$\Rightarrow \boxed{R_g + R_L = 2R_L} \Rightarrow \boxed{R_L = R_g}$$

(1)

1. The maximum power transfer theorem states that the maximum power is transferred from the source to the load when the load impedance is equal to the complex conjugate of the source impedance.

Let the source impedance be $Z_s = R_s + jX_s$ and the load impedance be $Z_L = R_L + jX_L$. The average power delivered to the load is given by:

$$P = \frac{1}{2} I_{rms}^2 R_L$$

where I_{rms} is the rms value of the current through the load. The current I is given by:

$$I = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + R_L + j(X_s + X_L)}$$

The magnitude of the current is:

$$|I| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

Substituting this into the power equation, we get:

$$P = \frac{1}{2} \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

To find the maximum power, we differentiate P with respect to R_L and X_L and set the derivatives to zero. For X_L , we get:

$$\frac{\partial P}{\partial X_L} = \frac{1}{2} \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \cdot \frac{-2(X_s + X_L)}{(R_s + R_L)^2 + (X_s + X_L)^2} = 0$$

This implies $X_s + X_L = 0$ or $X_L = -X_s$.

For R_L , we get:

$$\frac{\partial P}{\partial R_L} = \frac{1}{2} \frac{V_s^2 [(R_s + R_L)^2 + (X_s + X_L)^2] - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + (X_s + X_L)^2]^2} = 0$$

This simplifies to:

$$(R_s + R_L)^2 - R_L^2 = 0$$

which gives $R_L = R_s$.

Therefore, the maximum power is transferred when $Z_L = R_s - jX_s$, which is the complex conjugate of Z_s .

When $Z_L = Z_s^*$, the maximum power is:

$$P_{max} = \frac{1}{2} \frac{V_s^2 R_s}{(R_s + R_s)^2 + (X_s - X_s)^2} = \frac{1}{2} \frac{V_s^2 R_s}{4R_s^2} = \frac{1}{4} \frac{V_s^2}{R_s}$$

2. The maximum average power transfer theorem states that the maximum average power is transferred to a load when the load impedance is equal to the complex conjugate of the source impedance.

Let the source impedance be $Z_s = R_s + jX_s$ and the load impedance be $Z_L = R_L + jX_L$. The average power delivered to the load is given by:

$$P = \frac{1}{2} I_{rms}^2 R_L$$

where I_{rms} is the rms value of the current through the load. The current I is given by:

$$I = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + R_L + j(X_s + X_L)}$$

The magnitude of the current is:

$$|I| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

Substituting this into the power equation, we get:

$$P = \frac{1}{2} \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

To find the maximum power, we differentiate P with respect to R_L and X_L and set the derivatives to zero. For X_L , we get:

$$\frac{\partial P}{\partial X_L} = \frac{1}{2} \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \cdot \frac{-2(X_s + X_L)}{(R_s + R_L)^2 + (X_s + X_L)^2} = 0$$

This implies $X_s + X_L = 0$ or $X_L = -X_s$.

For R_L , we get:

$$\frac{\partial P}{\partial R_L} = \frac{1}{2} \frac{V_s^2 [(R_s + R_L)^2 + (X_s + X_L)^2] - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + (X_s + X_L)^2]^2} = 0$$

This simplifies to:

$$(R_s + R_L)^2 - R_L^2 = 0$$

which gives $R_L = R_s$.

Therefore, the maximum average power is transferred when $Z_L = R_s - jX_s$, which is the complex conjugate of Z_s .

3. The maximum average power transfer theorem states that the maximum average power is transferred to a load when the load impedance is equal to the complex conjugate of the source impedance.

Let the source impedance be $Z_s = R_s + jX_s$ and the load impedance be $Z_L = R_L + jX_L$. The average power delivered to the load is given by:

$$P = \frac{1}{2} I_{rms}^2 R_L$$

where I_{rms} is the rms value of the current through the load. The current I is given by:

$$I = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + R_L + j(X_s + X_L)}$$

The magnitude of the current is:

$$|I| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

Substituting this into the power equation, we get:

$$P = \frac{1}{2} \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$


Fig. 1.10

Solution:

- (i) Z_L is a complex impedance with both variable real and imaginary parts.

$$Z_L = 1 - j2 \Omega \quad (\text{Refer case ii})$$

$$\text{Load current} = I_L = \frac{10}{(1 + j2 + 1 - j2)} = 5 \text{ A.}$$

$$\text{Load power} = P_L = I_L^2 R_L = 5^2 \times 1 = 25 \text{ W}$$

- (ii) Z_L is a pure resistor $= R_L$

$$R_L = \sqrt{R_g^2 + X_g^2} = \sqrt{1^2 + 2^2} = 2.236 \Omega. \quad (\text{Refer case iv})$$

$$\text{Load Current} = I_L = \frac{10}{3.236 + j2} = 2.629 \text{ A,}$$

$$\text{Load power} = P_L = I_L^2 R_L = 15.45 \text{ W}$$

Problem 2 Find the value of Z_L is the previous problem, for maximum power transfer to it, given that Z_L can be varied only in magnitude.

Solution:

$$|Z_L| = \sqrt{1^2 + 2^2} = 2.236 \Omega \quad (\text{Refer case iii})$$

Problem 3 In the circuit shown in fig. 2.189 the load consists of a variable resistor and a capacitance variable between 10Ω and 20Ω . Find the value of R_L for maximum power transfer to the load and calculate the maximum power.

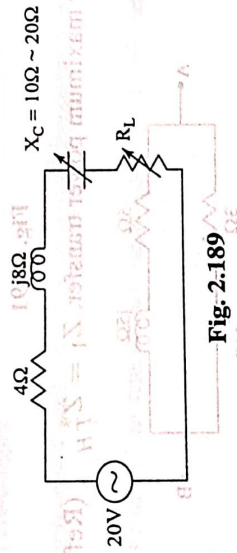


Fig. 2.189

Solution: For maximum power transfer,

$$Z_L = (4 - j8) \Omega \quad (\text{Refer case ii})$$

But since the value of load reactance can be varied only between 10Ω to 20Ω , $X_{\text{Load}} = -j10 \Omega$.

$$\text{Now, } R_g = 4 \Omega, X_g = j8 \Omega, X_L = -j10 \Omega.$$

$$\Rightarrow R_L = \sqrt{R_g^2 + (X_g + X_L)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = 4.472 \Omega. \quad (\text{Refer case vi})$$

$$\text{Load Current} = I_L = \frac{20}{(4 + j8 - j10 + 4.472)} = \frac{20}{\sqrt{8.472^2 + 2^2}} = 2.298 \text{ A}$$

Load Power, $P_L = I_L^2 R_L = 2.298^2 \times 4.472 = 23.607 \text{ W}$

Problem 4 Find the value of capacitive reactance to give maximum power transfer to the load in the circuit shown in fig. 2.190. Also find the power

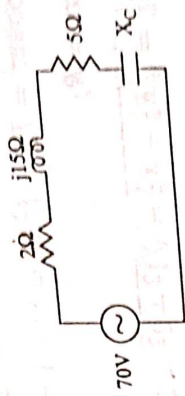


Fig. 2.190

Solution: For maximum power transfer to the load, $X_C = -j15 \Omega$. (Refer case ii)

$$\text{Load current} = I_L = \frac{70}{2 + j15 + 5 - j15} = 10 \text{ A.}$$

Power delivered to the load $= P_L = I_L^2 R_L = 10^2 \times 5 = 500 \text{ W}$

Problem 5 Find Z_L in the circuit shown, for maximum power transfer and find the maximum power.

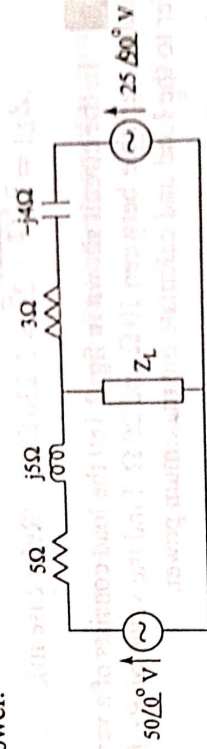


Fig. 2.191

Solution: For maximum power transfer, $Z_L = Z_T^*$ (Refer case ii)

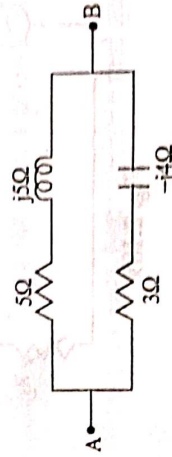


Fig. 2.192

$$Z_{TH} = (5 + j5) \parallel (3 - j4) = 4.385 \angle -15.255^\circ \Omega = (4.23 - j1.154) \Omega$$

Open circuit voltage (V_{AB}): (from fig. 2.193),

$$I = \frac{50 \angle 0^\circ - 25 \angle 90^\circ}{8 + j1} = 6.934 \angle -33.69^\circ \text{ A}$$

2.4 Rec

In a linear circuit, their posit

Problem theorem.

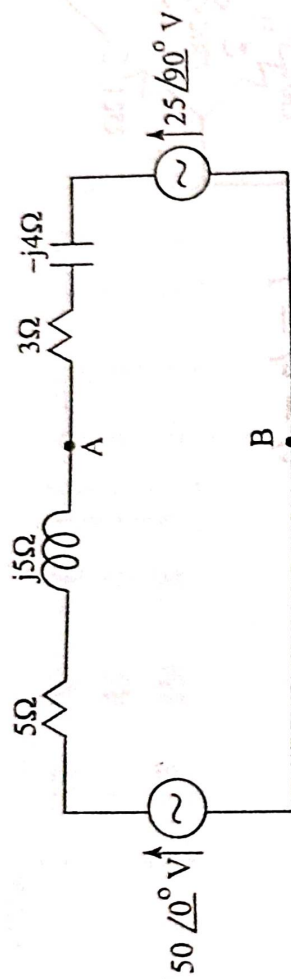


Fig. 2.193

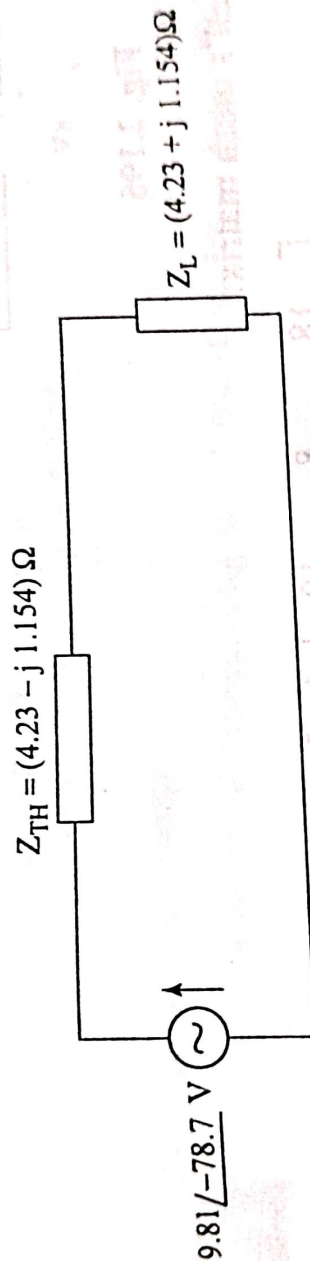


Fig. 2.194

$$V_{AB} = 50 - 6.934 \angle -33.69^\circ (5 + j5) = 9.81 \angle -78.7^\circ \text{ V}$$

Thevenin's equivalent circuit:

$$\text{Load current } = I_L = \frac{9.81 \angle -78.7^\circ}{4.23 + 4.23} = 1.1596 \angle -78.7^\circ \text{ A}$$

$$\text{Power delivered to the load } = P_L = I_L^2 R_L = 1.1596^2 \times 4.23$$

$$= 5.688 \text{ W}$$

EET 201

①

Circuits and Networks

↓
closed loop ← devices operate

Ohm's Law $\rightarrow V = IR$

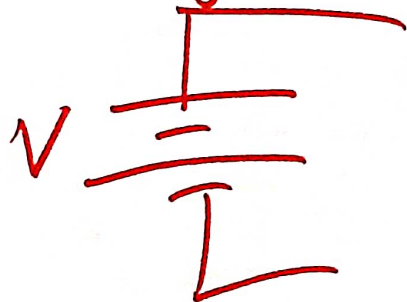
$$I = \frac{V}{R} \quad , \quad R = \frac{V}{I}$$

Network $\rightarrow$ energy source and elements (R, C, L)

↳ Active $\rightarrow$ V, I sources

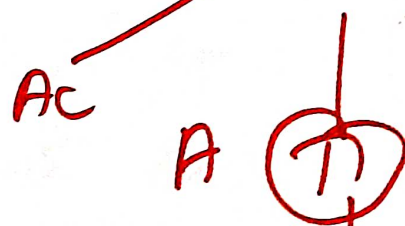
↳ Passive $\rightarrow$ R, C, L

Voltage source



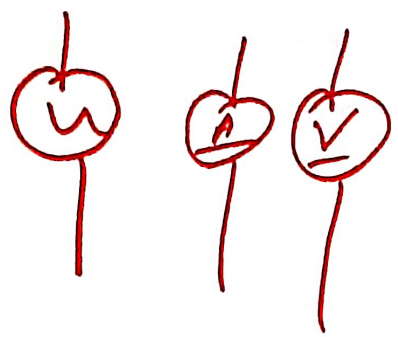
AC DC

DC Current source

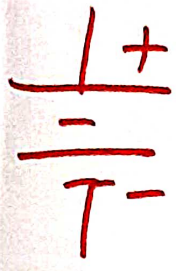


AC source - AC circuits
DC source - DC circuits

AC



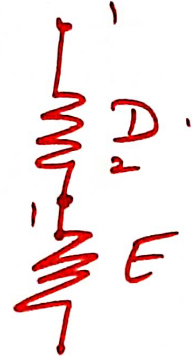
DC



Resistance in series



Same current
Different voltage



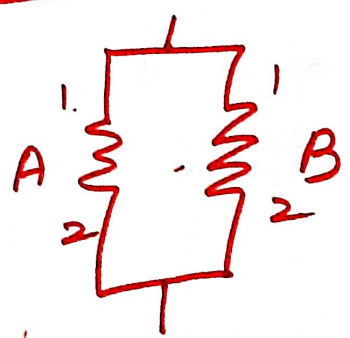
$$R_{eff} = R_1 + R_2 + R_3 + \dots$$

$$V_1 = IR_1$$

$$V_2 = IR_2$$

Current flows thro' the element
Voltage appear across the element

Resistance in parallel



Same Voltage
Different current

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_{eq}}$$

Total current = I

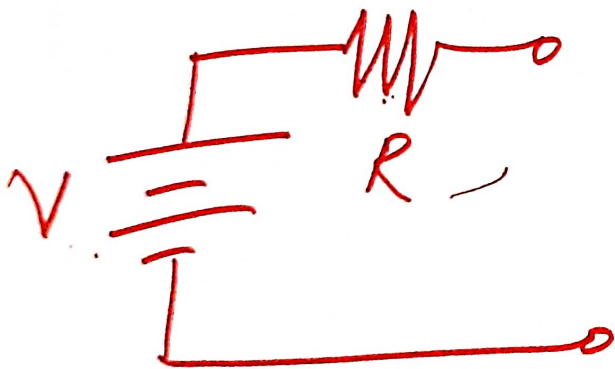
(3)

$$I_1 = \frac{I R_2}{R_1 + R_2}$$

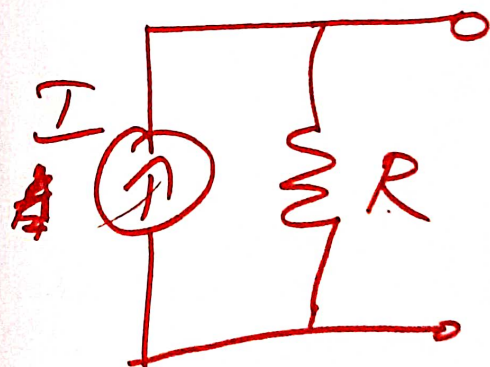
$$I_2 = \frac{I R_1}{R_1 + R_2}$$

Source Transformation

Convert V source into I source
and vice versa



V source in series
with Resistor



Current source
in parallel with
Resistor.

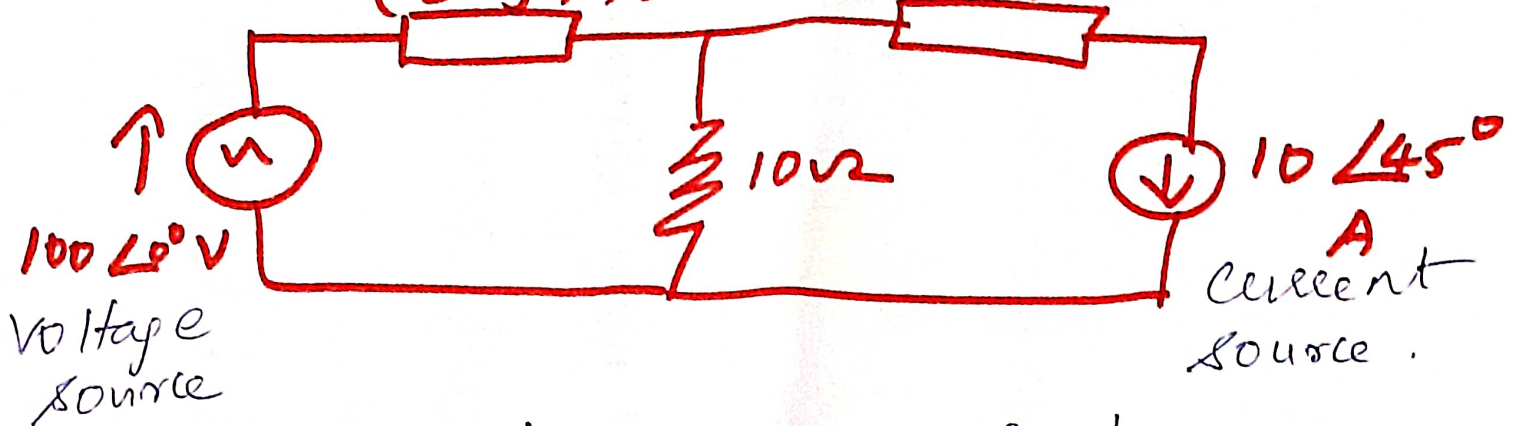
KCL - AC circuits

①

All current sources

$(3+j4)\Omega$

$(4-j3)\Omega$



Convert voltage source into current source.

$$I = \frac{V}{R(\Omega)} = \frac{100 \angle 0^\circ}{3+j4} = \frac{100 \angle 0^\circ}{5 \angle 53.13^\circ}$$

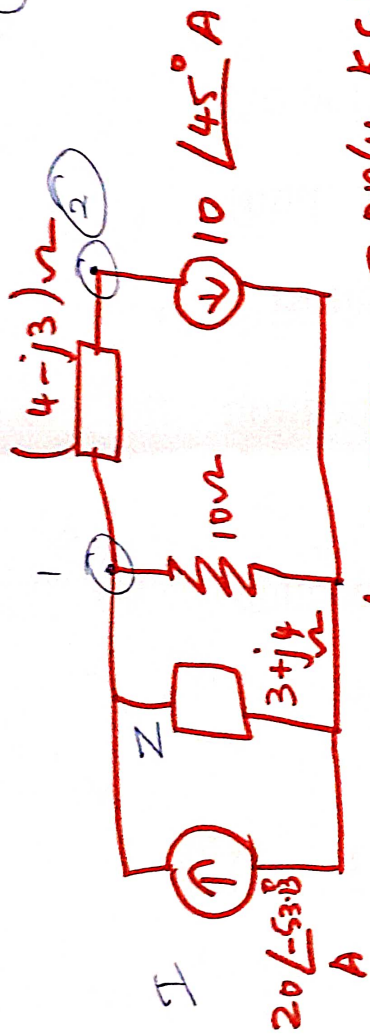
$$3+j4 = 5 \angle 53.13^\circ$$

$$r = \sqrt{A^2+B^2} = \sqrt{3^2+4^2} = 5$$

$$\theta = \tan^{-1}(B/A) = \tan^{-1}(4/3) = 53.13^\circ$$

$$= 20 \angle 0 - 53.13 = 20 \angle -53.13^\circ \text{ A.}$$

2



All current sources - apply KCL

Current source parallel with 2

$$\left| \frac{1}{2} \right| |V| = |I|$$

$$\begin{cases} \frac{1}{3+j4} + \frac{1}{10} + \frac{1}{4-j3} (-\frac{1}{4-j3}) & |V_1| \\ (-)\frac{1}{4-j3} & |V_2| \end{cases} = \begin{cases} \frac{20}{\angle -53.8^\circ} \\ -10 \\ \angle 45^\circ \end{cases}$$

Taking LCM

$$\frac{1}{3+j4} + \frac{1}{10} + \frac{1}{4-j3}$$

$$\frac{[(3+j4)(4-j3)] + [(10)(4-j3)] + [(3+j4)(10)]}{(3+j4)(10)(4-j3)}$$

3

$$\frac{(12 - j9 + j16 - j^2 12) + (40 - j30) + (30 + j40)}{(30 + j40)(4 - j3)}$$

$$\frac{(12 + j7 + 12) + (40 - j30) + (30 + j40)}{120 - j90 + j160 - j^2 120}$$

$$\frac{94 + j17}{240 + j70} = \frac{95.42 \angle 10.25^\circ}{250 \angle 16.26^\circ} = 0.38 \angle -6.01^\circ$$

$$\begin{aligned} \frac{94 + j17}{\sqrt{94^2 + 17^2}} &= \frac{95.42}{\sqrt{9402 + 702}} = \frac{95.42}{250} = 0.38 \\ \tan^{-1}\left(\frac{17}{94}\right) &= 10.25^\circ \\ 4 - j3 &= 5 \angle -36.87^\circ \end{aligned}$$

$$\begin{aligned} \frac{1}{4 - j3} &= \frac{1}{5 \angle -36.87^\circ} = \frac{0.2 \angle 36.87^\circ}{5} = 0.04 \angle 36.87^\circ \\ \frac{1}{4 - j3} &= \frac{1}{5 \angle -36.87^\circ} = 0.2 \angle 36.87^\circ \end{aligned}$$

$$\frac{1}{2} \begin{bmatrix} 0.38 \angle -6.01^\circ & -0.2 \angle 81.87^\circ \\ -0.2 \angle 81.87^\circ & 0.2 \angle 81.87^\circ \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \angle -53.13^\circ \\ -10 \angle 45^\circ \end{bmatrix}$$

$$\begin{aligned} \Delta &= (0.38 \times 0.2 \angle -6.01^\circ + 81.87^\circ) - \\ &\quad (-0.2 \times -0.2 \angle 81.87^\circ + 81.87^\circ) \\ &= (0.076 \angle 75.86^\circ) - (0.04 \angle 43.74^\circ) \\ &= (0.019 + j0.074) - (-0.038 + j0.011) \\ &= 0.019 + j0.074 + 0.038 - j0.011 \\ \Delta &= 0.057 + j0.063 \quad \text{--- rectangular form} \\ \Delta &= 0.085 \angle 47.86^\circ \quad \text{--- polar form} \end{aligned}$$

$$\Delta V_1 = \begin{vmatrix} 20 \angle -53.13 & -0.2 \angle 81.87 \\ -10 \angle 45 & 0.2 \angle 81.87 \end{vmatrix}$$

$$= (20 \times 0.2 \angle -53.13 + 81.87) -$$

$$(-0.2 \times -10 \angle 81.87 + 45)$$

$$= (4 \angle 28.74) - (2 \angle 126.87)$$

$$= (3.51 + j 1.92) - (-1.20 + 1.5j)$$

$$= 3.51 + j 1.92 + 1.20 - j 1.6$$

$$= 4.71 + 0.32j = \Delta V_1 - \text{rectangular form}$$

$$= 4.751 \angle 3.887 = \Delta V_1 = \text{polar form.}$$

By Cramer's Rule

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{4.751 \angle 3.887}{0.085 \angle 47.86}$$

$$= 55.89 \angle 3.887 - 47.86$$

$$= 55.89 \angle -43.973 \text{ V} = \text{polar form}$$

6

$$V_1 = 40 \angle -22^\circ - j 38.81 V \Rightarrow \text{Rectangular form}$$

$$\Delta V_2 = \begin{vmatrix} 0.38 \angle -6.01^\circ & 20 \angle -53.13^\circ \\ -0.2 \angle 81.87^\circ & -10 \angle 45^\circ \end{vmatrix}$$

$$= (0.38 \times -10 \angle -6.01^\circ + 45) -$$

$$(20 \times -0.2 \angle 81.87^\circ + (-53.13))$$

$$= (-3.8 \angle 38.99^\circ) - (-4 \angle 28.74^\circ)$$

$$= \cancel{39.17}$$

$$= (-2.95 - j 2.39) - (-3.51 - 1.92j)$$

$$= -2.95 - j 2.39 + 3.51 + j 1.92$$

$$= 0.56 - j 0.47 = \Delta V_2 = \text{Rectangular}$$

$$= 0.73 \angle -40.006^\circ = \Delta V_2 = \text{Polar}$$

$$V_2 = \frac{\Delta V_2}{\Delta} = \frac{0.73 \angle -40.006^\circ}{0.085 \angle 47.86^\circ}$$

$$= 8.588 \angle -40.006 - 47.86$$

(7)

$$V_2 = 8.588 \angle -87.866 \text{ V} = \text{polar form}$$

$$V_2 = 0.3198 - j 8.582 \text{ V} = \text{Rectangular form}$$

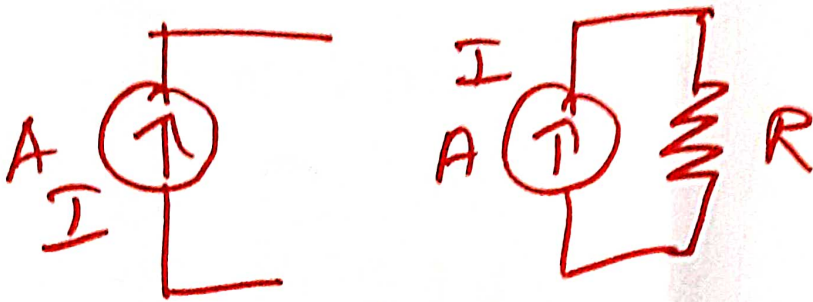
DC circuit

Kirchoff Current Law (KCL) 1

Total

Current at a junction is zero
Incoming current = outgoing
current in a node

Apply KCL, all current sources



Identify Nodes.

Node = a junction wherein two
or more elements join together

No. of nodes = order of matrix

eg. $2 = 2 \times 2$

$3 = 3 \times 3$

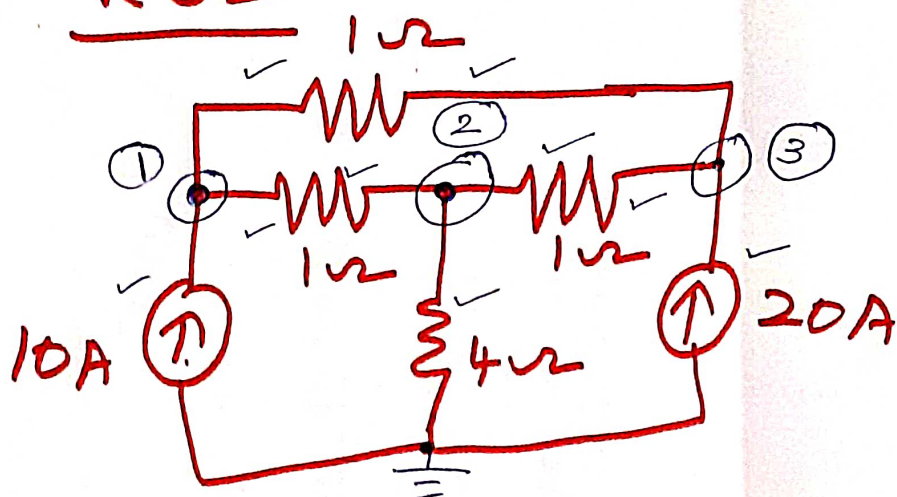
Each and every element will
be connected to two different
nodes

Select minimum no. of nodes

KCL $\rightarrow$ Nodal voltages.

KCL

All current sources (2)



No. of nodes = 3 = order of matrix
= 3×3

Ohm Law = $V = IR$

$$I = \frac{V}{R}$$

$$|I| = \left| \frac{1}{R} \right| |V|$$

$$\left| \frac{1}{R} \right| |V| = |I|$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{1} + \frac{1}{1} & -\frac{1}{1} & -\frac{1}{1} \\ -\frac{1}{1} & \frac{1}{1} + \frac{1}{1} + \frac{1}{4} & -\frac{1}{1} \\ -\frac{1}{1} & -\frac{1}{1} & \frac{1}{1} + \frac{1}{1} \end{vmatrix}$$

At node 1
 $a_{11} = \left| \frac{1}{1} + \frac{1}{1} \right| = 2$

At node 2
 $a_{22} = \frac{1}{1} + \frac{1}{1} + \frac{1}{4} = 2.25$

At node 3
 $a_{33} = \frac{1}{1} + \frac{1}{1}$

$a_{12} = a_{21} = R$ bet node 1 and 2
 $a_{32} = a_{23} = R$ bet node 2 and 3

$$\left| \frac{1}{R} \right| \left| V \right| = \left| I \right|$$

(3)

↳ Nodal Voltage

$$\begin{vmatrix} 2 & -1 & -1 \\ -1 & 2.25 & -1 \\ -1 & -1 & 2 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix}$$

V_1 = voltage at node 1

V_2, V_3 = V at node 2 and 3.

For all common elements ($a_{12}, a_{21}, a_{31}, a_{13}, a_{23}, a_{32}$) put negative sign

for current sources

$\uparrow \uparrow \Rightarrow$ — put positive sign

$\downarrow \downarrow, \Leftarrow$ } put negative sign

$$V_1 = 133.33 \text{ V}$$

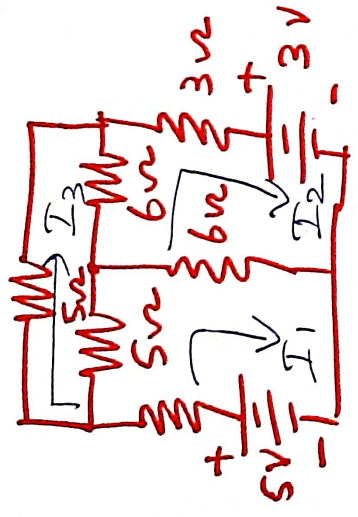
$$V_2 = 120 \text{ V}$$

$$V_3 = 136.66 \text{ V}$$

KVL - ~~10V~~ ^{5V} voltage source

(5)

$$\begin{aligned}
 a_{11} &= 5 + 5 + 6 = 16 \\
 a_{22} &= 6 + 6 + 3 = 15 \\
 a_{33} &= 5 + 10 + 6 = 21 \\
 a_{21} &= a_{12} = 6 \\
 a_{13} &= a_{31} = 5 \\
 a_{23} &= a_{32} = 6
 \end{aligned}$$



$$|R| |I| = |V|$$

$$\begin{vmatrix} 16 & -6 & -5 \\ -6 & 15 & -6 \\ -5 & -6 & 21 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 3 \\ 0 \end{vmatrix}$$

$I_1, I_2 \Rightarrow$ same, so -5.

$\rightarrow I_1 \Rightarrow$ same, so -5
 $\rightarrow I_3$

$\rightarrow I_2 \Rightarrow$ same, so -6
 $\rightarrow I_3$

$$\begin{aligned}
 I_1 &= A \\
 I_2 &= A \\
 I_3 &= A
 \end{aligned}$$

Ohm Law $\rightarrow$ source transformation

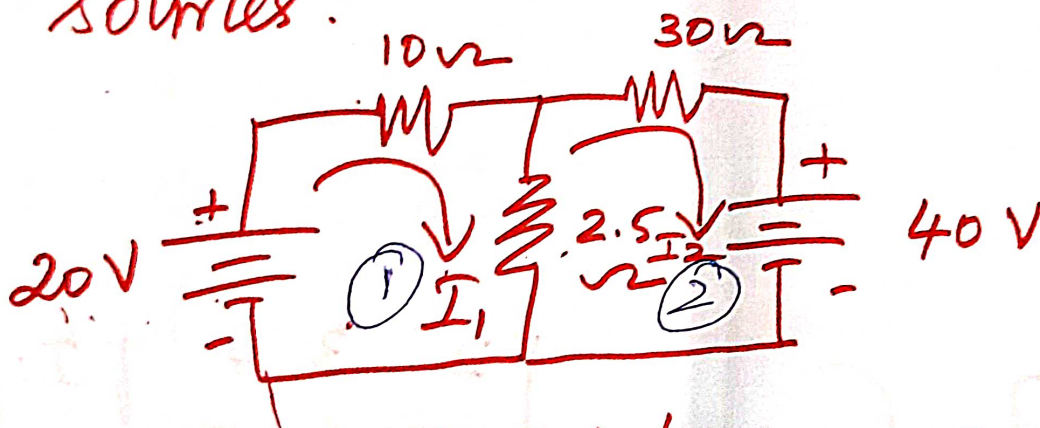
$$I = \frac{V}{R} \quad , \quad V = IR$$

①

Kirchoff's Voltage Law : DC circuit

Algebraic sum of voltage drops in a closed loop is zero.

Using KVL $\Rightarrow$ calculate ^{loop} current
Apply KVL $\Rightarrow$ circuits having voltage sources.



Matrix Method.

All Voltage sources.

No. of closed loop = 2

Order of matrix = 2×2 $\leftarrow$ columns

No. of closed loop = $\overset{\text{rows}}{\text{order of}}$ matrix.

$$\begin{array}{c}
 \begin{array}{cc} a_{11} & a_{12} \\ \left| \begin{array}{cc} 12.5 & -2.5 \\ -2.5 & 32.5 \end{array} \right| \end{array} \begin{array}{c} I_1 \\ I_2 \end{array} = \begin{array}{c} \textcircled{2} \quad \text{same} \\ \left| \begin{array}{c} 20 \\ 40 \end{array} \right| \end{array} \begin{array}{c} \downarrow \quad \downarrow \\ I_1 \quad I_2 \\ (-ve) \end{array} \\
 \text{Ohm's Law: } \left| R \right| \begin{array}{c} a_{21} \quad a_{22} \\ \left| \begin{array}{cc} 12.5 & -2.5 \\ -2.5 & 32.5 \end{array} \right| \end{array} \begin{array}{c} I_1 \\ I_2 \end{array} = \begin{array}{c} \left| V \right| \\ \left| \begin{array}{c} 20 \\ -40 \end{array} \right| \end{array}
 \end{array}$$

a_{11} = Sum of R in loop 1 = $10 + 2.5$

a_{22} = Sum of R in loop 2 = $30 + 2.5$

$a_{12} = a_{21}$ = Common R to loop
 $\textcircled{1}$ and $\textcircled{2}$ = 2.5

Based on loop current direction,
 assign sign to common
 element a_{12}, a_{21} (2.5 Ω)

$\therefore$ direction of I_1 and I_2 are
 same ($\downarrow I_1, \downarrow I_2$), use negative
 sign (-2.5)

3

Based on loop current leaving terminal, we polarity for voltage source.

I_1 is leaving positive terminal of 20V. ($\therefore$ using +20V)

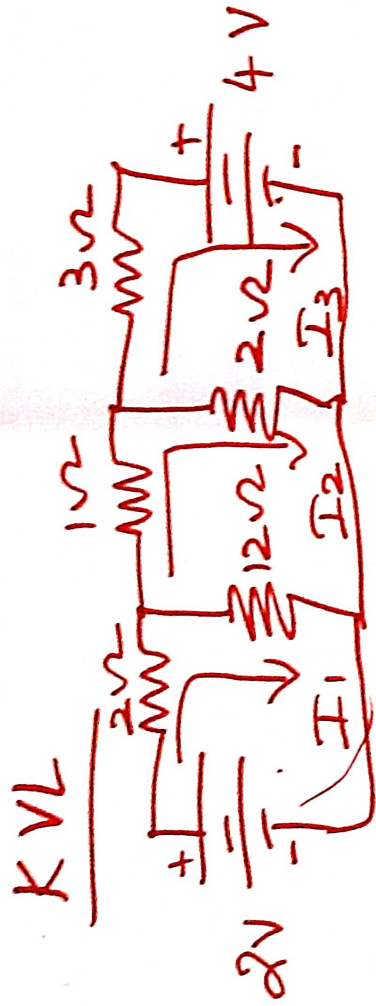
I_2 is leaving negative terminal of 40V. ($\therefore$ using -40V)

$$\left. \begin{aligned} 12.5 I_1 - 2.5 I_2 &= 20 \\ -2.5 I_1 + 32.5 I_2 &= -40 \end{aligned} \right\}$$

from matrix, write loop equation

$$I_1 = 1.375 \text{ A}, I_2 = -1.125 \text{ A}.$$

KVL



All voltage sources

No. of closed loop = 3

Order of matrix = 3×3

Draw loop current

~~HR~~ $|R||I| = |V| \Rightarrow \text{Ohm Law}$

$$a_{11} = 2 + 12 = 14\Omega \quad a_{12} = a_{21} = 12$$

$$a_{22} = 12 + 1 + 2 = 15\Omega$$

$$a_{33} = 3 + 2 = 5\Omega \quad a_{13} = a_{31} = 0$$

$$a_{23} = a_{32} = 2$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

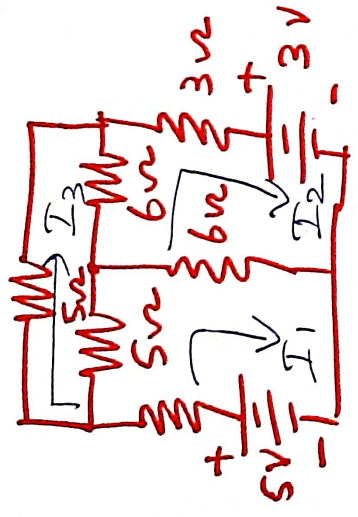
$R \quad \quad \quad V$

$$I_1 = 0.16 \text{ A} \quad I_2 = 0.02 \text{ A} \quad I_3 = -0.78 \text{ A}$$

KVL - ~~10V~~ ^{5V} voltage source

(5)

$$\begin{aligned}
 a_{11} &= 5 + 5 + 6 = 16 \\
 a_{22} &= 6 + 6 + 3 = 15 \\
 a_{33} &= 5 + 10 + 6 = 21 \\
 a_{21} &= a_{12} = 6 \\
 a_{13} &= a_{31} = 5 \\
 a_{23} &= a_{32} = 6
 \end{aligned}$$



$$|R| |I| = |V|$$

$$\begin{vmatrix} 16 & -6 & -5 \\ -6 & 15 & -6 \\ -5 & -6 & 21 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 3 \\ 0 \end{vmatrix}$$

$I_1, I_2 \Rightarrow$ same, so -5.

$\rightarrow I_1 \Rightarrow$ same, so -5
 $\rightarrow I_3$

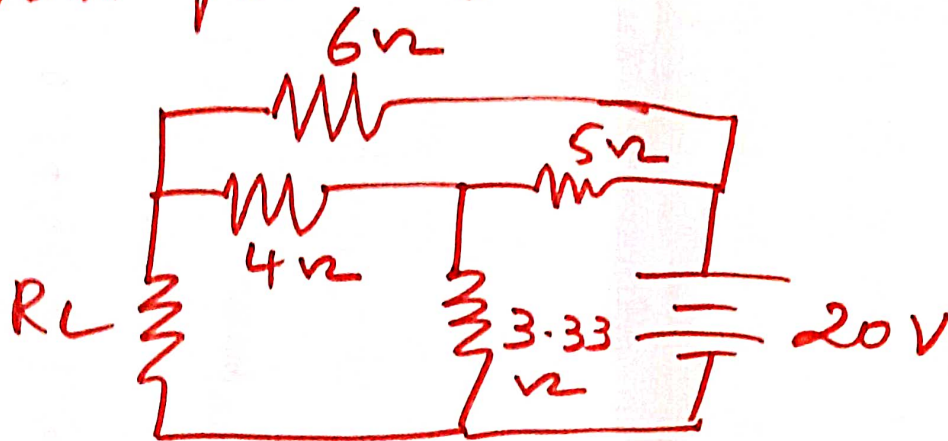
$\rightarrow I_2 \Rightarrow$ same, so -6
 $\rightarrow I_3$

$$\begin{aligned}
 I_1 &= A \\
 I_2 &= A \\
 I_3 &= A
 \end{aligned}$$

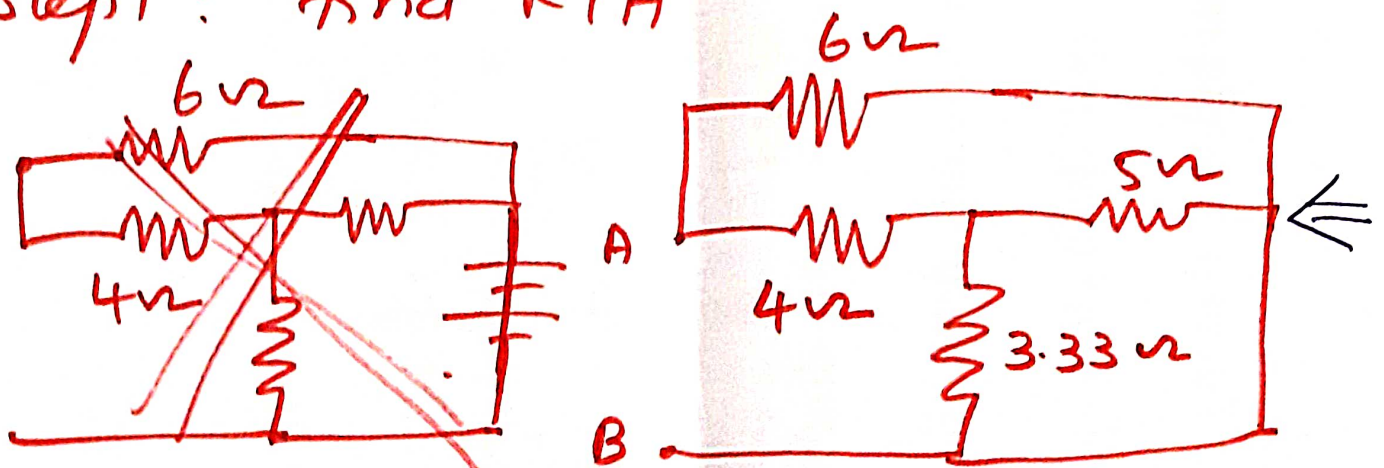
Maximum Power Transfer Theorem. DC circuits (1)

$$R_L = R_{TH}$$

Find R_L for which maximum power is transferred. Find the max. power delivered.



Step 1: Find R_{TH}



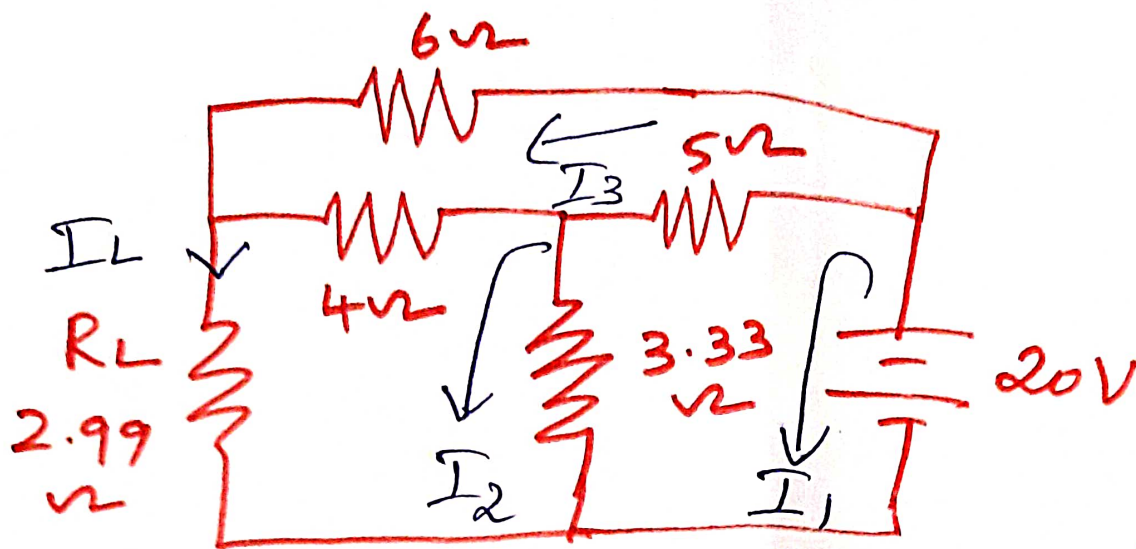
Find R_{TH} across AB.

$$R_{TH} = 2.99\Omega = R_L$$

Power delivered (power delivered by load resistance)

$$P_L = I_L^2 R_L$$

(2)



I_L is current flowing thro' R_L
Use KVL to find I_L

$$\begin{vmatrix} 8.33 & -3.33 & -5 \\ -3.33 & 10.32 & -4 \\ -5 & 4 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 20 \\ 0 \\ 0 \end{vmatrix}$$

$$I_2 = 2.33 \text{ A.} = I_L$$

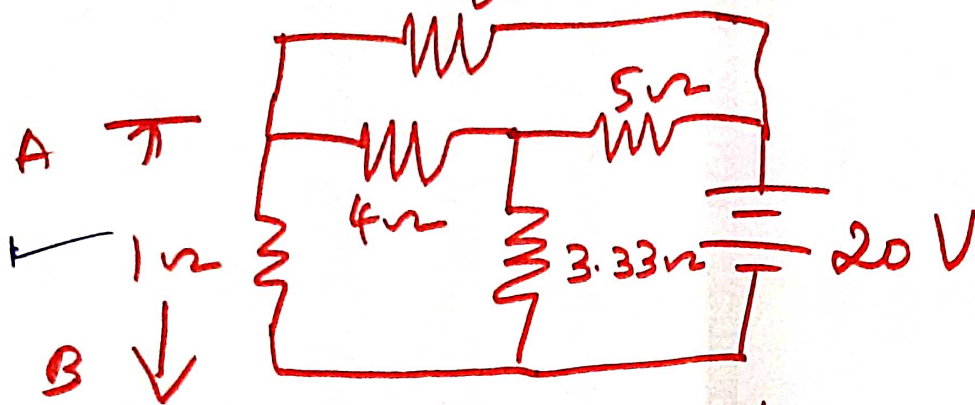
$$P_L = I_L^2 R_L = (2.33)^2 (2.99) = 16.23 \text{ Watt.}$$

DC circuits

Norton's Theorem

①

Find the current thro' 1Ω Resistor in the circuit using Norton Theorem

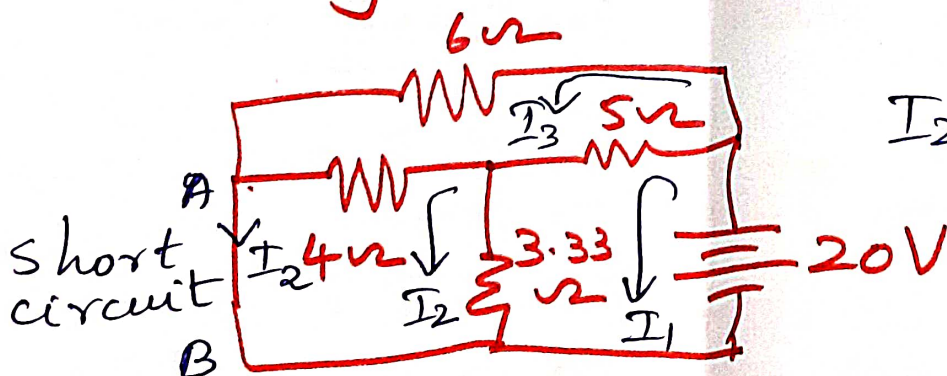


1. Apply Norton Theorem across AB
2. Find Load Resistance $R_L = 1\Omega$ across AB.

3. Find R_{TH}

$$R_{TH} = 2.99\Omega$$

4. Find I_{SC} : Short circuit current through AB (Norton Terminals)



I_2 is flowing thro' AB
 $\therefore I_2$ is short circuit current.

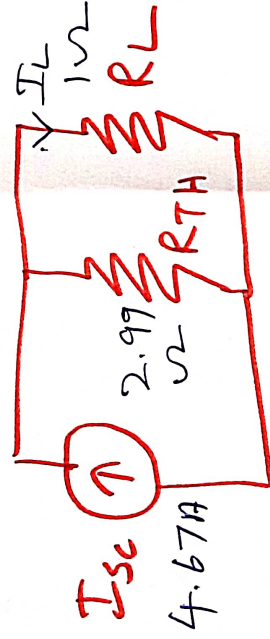
(2)

By KVL

$$\begin{bmatrix} 8.33 & -3.33 & -5 \\ -3.33 & 7.33 & -4 \\ -5 & -4 & 16.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

$$I_2 = 4.67 \text{ A} = I_{SC}$$

5. Draw Norton Equivalent circuit



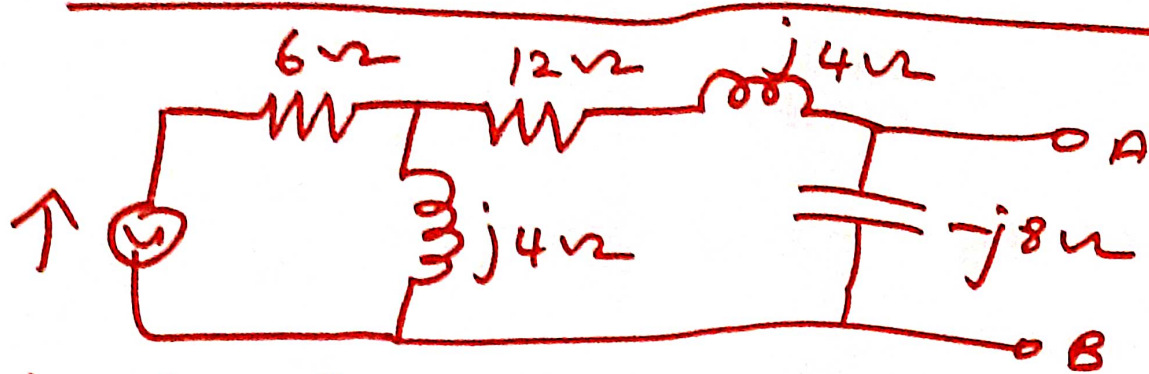
If I_{sc} is +ve $\uparrow I_{sc}$

If I_{sc} is -ve $\downarrow I_{sc}$

6. Find current thru' $1\Omega R = I_L$

$$\text{Using Current Division Rule} \quad I_L = \frac{I_{sc} R_{TH}}{R_{TH} + R_L} = \frac{4.67 \times 2.99}{2.99 + 1} = 3.5 \text{ A}$$

Norton Theorem - AC circuit



$$120 \angle 45^\circ \text{ V}$$

No Load Resistance / Impedance.
No element connected between
A and B

A B $\Rightarrow$ Norton Terminal.

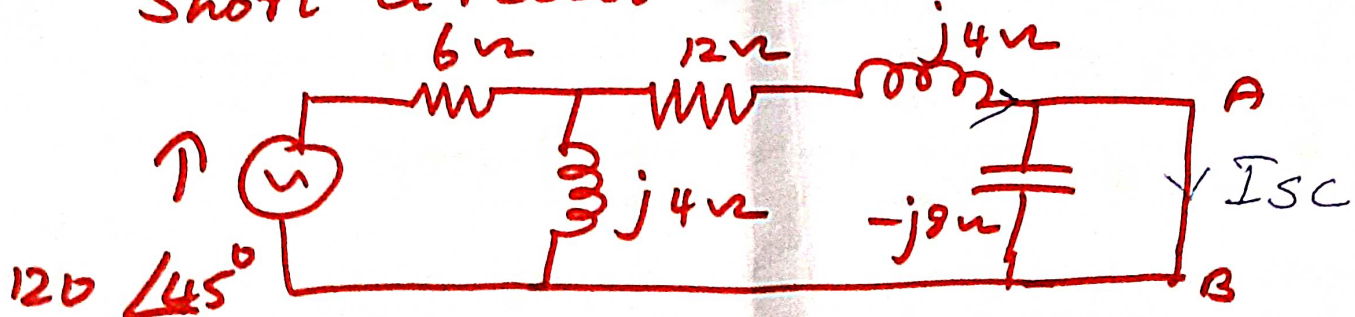
Step 1 : calculate Z_{TH}

$$Z_{TH} = 3.74 - j6.511 \Omega$$

$$Z_{TH} = 7.51 \angle -60.13^\circ \Omega$$

Step 2 : calculate I_{SC}

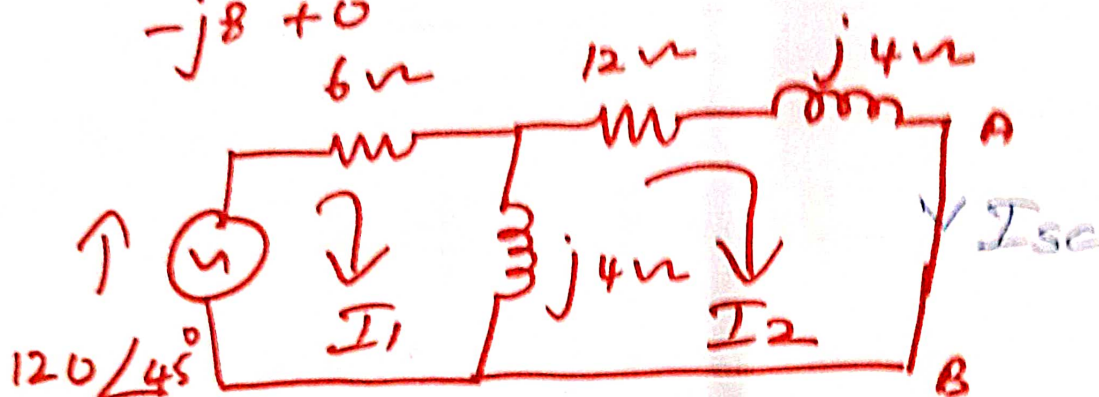
Short circuit current thro' A B



Since no element between A & B,
All currents will flow thro' AB.

$-j8\Omega$ // AB path
 Resistance / Impedance in AB path
 is zero.
 parallel combination $-j8\Omega$ and AB
 is zero

$$\frac{-j8 \times 0}{-j8 + 0} = 0$$



$\checkmark$ $I_{sc} = I_2$

$\therefore$ calculate I_2 , use KVL

$$|Z| |I| = |V| \Rightarrow (2 \times 2) \text{ matrix}$$

$$\begin{vmatrix} 6+j4 & -j4 \\ -j4 & 12+j8 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 120\angle 45^\circ \\ 0 \end{vmatrix}$$

$$\Delta = [(6+j4)(12+j8)] - (-j4 \times -j4)$$

$$= (72 + j48 + j48 + j^2 32) - (j^2 16)$$

$$\begin{aligned}
 &= (72 + j96 - 32) - (-16) \\
 &= 40 + j96 + 16 = 56 + j96 = \Delta \\
 \Delta &= \frac{\sqrt{56^2 + 96^2}}{\tan^{-1}\left(\frac{96}{56}\right)} = 111.14 \angle 59.74^\circ
 \end{aligned}$$

$$\Delta I_2 = \begin{vmatrix} 6 + j4 & 120 \angle 45^\circ \\ -j4 & 0 \end{vmatrix}$$

$$\begin{aligned}
 120 \angle 45^\circ &= 120 \cos 45^\circ + j 120 \sin 45^\circ \\
 &= 84.85 + j 84.85
 \end{aligned}$$

$$\Delta I_2 = \begin{vmatrix} 6 + j4 & 84.85 + j 84.85 \\ -j4 & 0 \end{vmatrix}$$

$$= (6 + j4) \cdot 0 - (-j4(84.85 + j 84.85))$$

$$= -(-j 339.4 - j^2 339.4)$$

$$= -(-j 339.4 + 339.4)$$

$$= j 339.4 - 339.4 = \Delta I_2$$

$$= -339.4 + j 339.4$$

$$= \frac{\sqrt{339.4^2 + 339.4^2}}{\tan^{-1}\left(\frac{339.4}{-339.4}\right)}$$

$$= 479.98 \angle -45^\circ = \Delta I_2$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{479.98 \angle -45^\circ}{111.14 \angle 59.74^\circ}$$

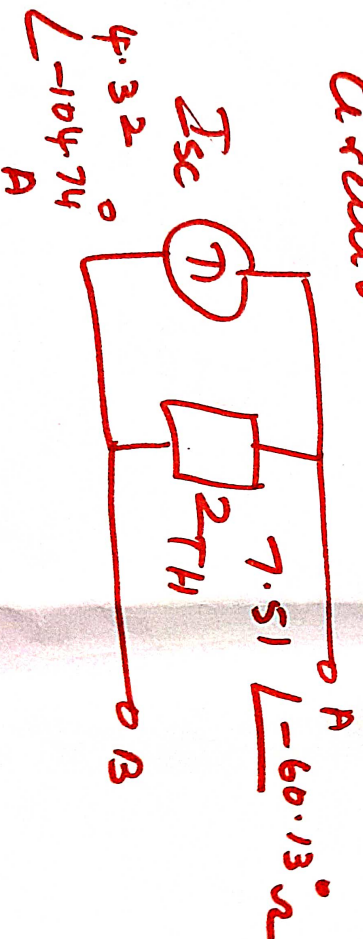
$$= 4.32 \angle -104.74^\circ \text{ A}$$

$$= 4.32 \cos(-104.74^\circ) + j 4.32 \sin(-104.74^\circ)$$

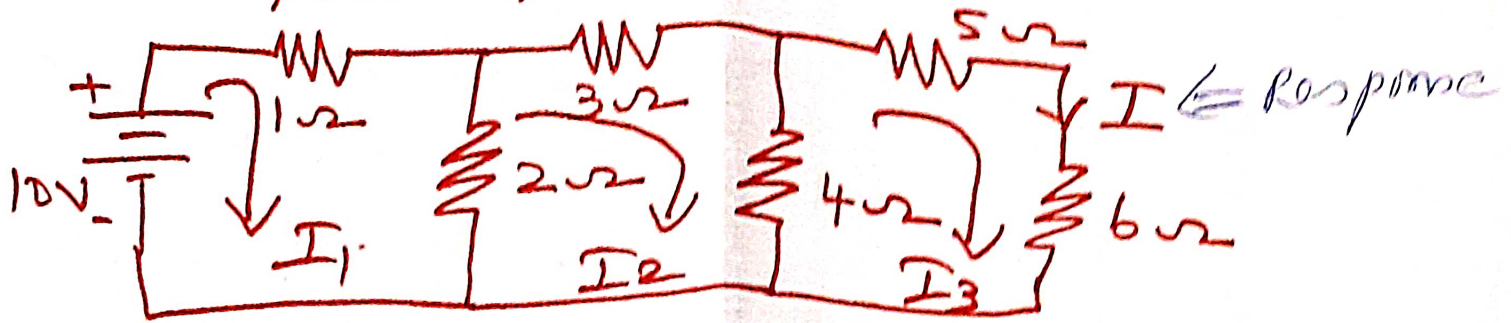
$$= -1.099 - j 4.178 \text{ A}$$

$$I_2 = I_{sc}$$

step 3 : Draw Norton Equivalent circuit



Reciprocity Theorem Dc circuit



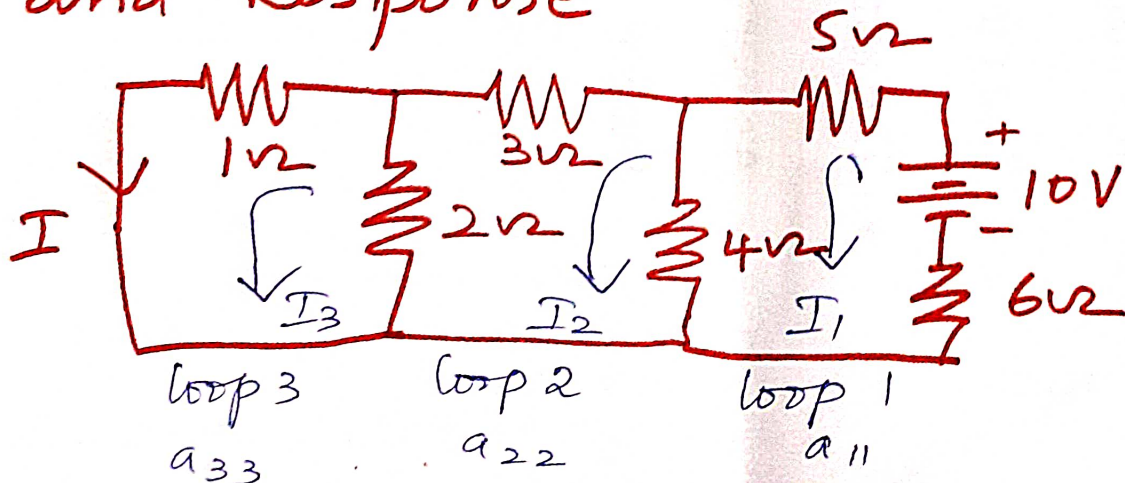
1. Draw loop current
2. Identify Input (Excitation) = 10 V
3. Identify output (Response) = I
4. Calculate output (I)
5. $I_3 = I$

Use KVL (R)

$$\begin{vmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 0 \\ 0 \end{vmatrix}$$

$$I_3 = 0.27 \text{ A}$$

Interchange position of excitation and Response



Input = 10 V, o/p = $I = I_3$

(2)

Use KVL

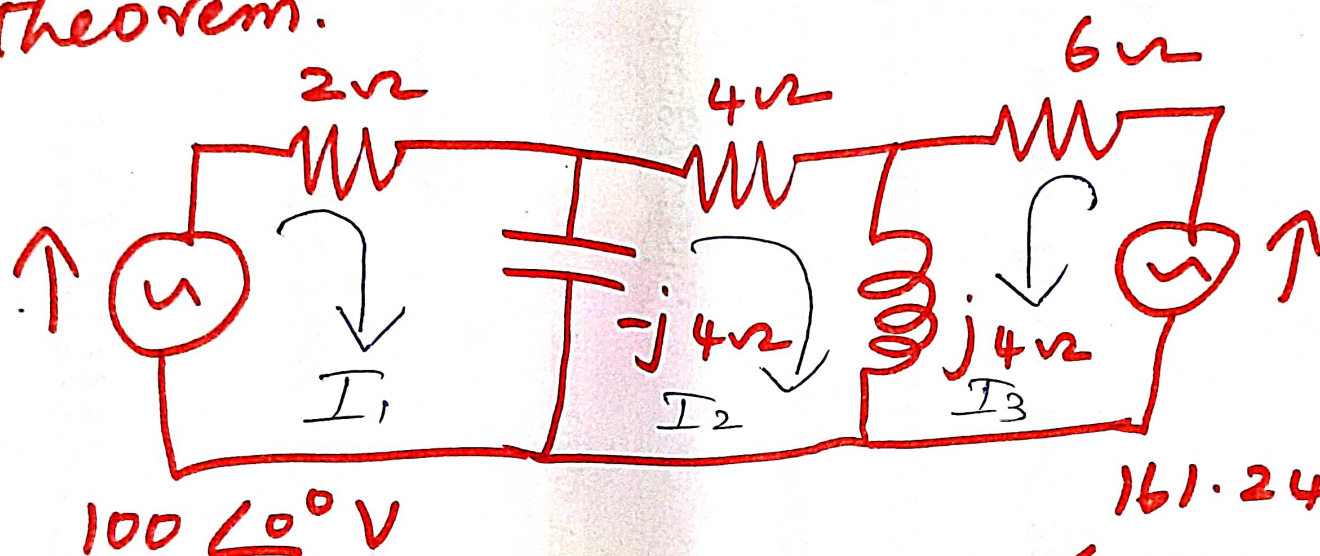
$$\begin{vmatrix} 15 & -4 & 0 \\ -4 & 9 & -2 \\ 0 & -2 & 3 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 0 \\ 0 \end{vmatrix}$$

$$I_3 = 0.27 \text{ A}$$

If we interchange the position of excitation and response, we will get same answer.

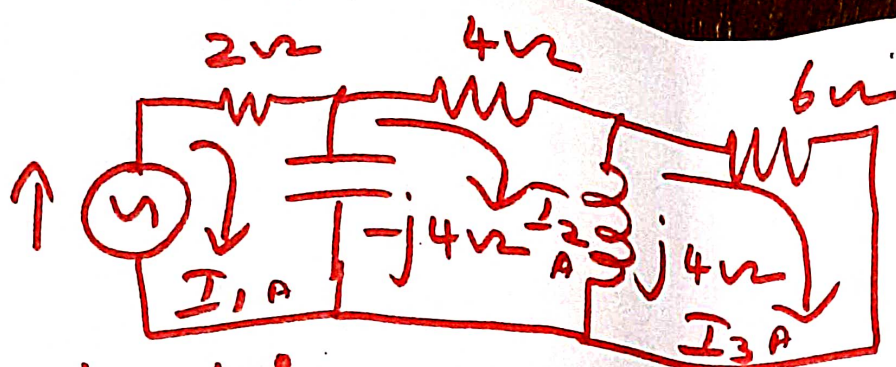
Superposition Theorem AC circuits

Find current thro' $4\Omega R$ in the given circuit by superposition theorem.



Step 1: Draw loop currents
Current will flow from the arrow ↑

Step 2: only one source acting at one time
only 100∠0° V
Short circuit 161.245 ∠-82.9° V



$$100 \angle 0^\circ \text{ V}$$

Draw loop currents based on $100 \angle 0^\circ \text{ V}$
 Three loop currents = Three closed loops = (3×3) matrix

(R)

$$\begin{pmatrix} 2 - j4 & -j4 & 0 \\ -j4 & 4 + j4 - j4 & (-)j4 \\ 0 & (-)j4 & 6 + j4 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 100 \angle 0^\circ \\ 0 \\ 0 \end{pmatrix}$$

$(-)$ Same loop current direction
 0 same loop current direction
 $(-)$ $j4$ direction

$$\begin{pmatrix} 2 - j4 & j4 & 0 \\ j4 & 4 & -j4 \\ 0 & -j4 & 6 + j4 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 100 \angle 0^\circ \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta = 2 - j4 \left[4(6 + j4) - (-j4 \times -j4) \right] - j4 \left[j4(6 + j4) - (0 \times -j4) \right] + 0$$

$j^2 = -1$

(3)

$$= 2 - j4 [24 + 16j + 16] - j4 [j24 - 16]$$

$$= 48 + 32j + 32 - j96 + 64 - j64 + 96 + j64$$

$$= 240 - j64 = \Delta$$

$$= 248.39 \angle -14.93^\circ$$

Current thro' 4Ω is I_2

$$\Delta I_2 = \begin{vmatrix} 2 - j4 & 100 \angle 0^\circ & 0 \\ j4 & 0 & -j4 \\ 0 & 0 & 6 + j4 \end{vmatrix}$$

$$100 \angle 0^\circ = 100 + j0$$

$$= \begin{vmatrix} 2 - j4 & 100 + j0 & 0 \\ j4 & 0 & -j4 \\ 0 & 0 & 6 + j4 \end{vmatrix}$$

$$= 2 - j4 [0] - [100 + j0 (j4 (6 + j4))] + 0$$

$$= -j400 + j$$

(14)

$$= -100(j24 - 16) = -j2400 + 1600 = 0I_2$$

$$0I_2 = 2884.44 \angle -56.31$$

By Cramer's Rule

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{2884.44 \angle -56.31}{248.39 \angle -14.93}$$

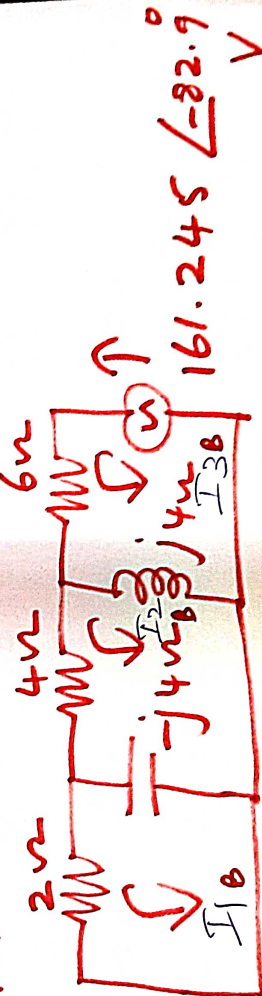
$$= 11.61 \angle -56.31 - (-14.93)$$

$$I_{2A} = 11.61 \angle -41.38 \text{ A}$$

$$I_{2A} = 8.711 - j7.67 \text{ A}$$

Step 3: $161.245 \angle -82.9^\circ \text{ V alone}$

short circuit $100 \angle 0^\circ \text{ V}$



Draw loop current based on
 $161.245 \angle -82.9^\circ \text{ V} \uparrow$

$$\underline{Z} \quad \frac{1}{Z} = \frac{1}{V} \quad \textcircled{5}$$

$$\frac{1}{I} = \frac{1}{V}$$

$$\begin{pmatrix} 2-j4 & (-j4) & 0 \\ (-j4) & 4-j4+j4 & j4 \\ 0 & (-j4) & 6+j4 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 161.245 \angle -82.9^\circ \end{pmatrix}$$

$$\begin{pmatrix} 2-j4 & j4 & 0 \\ j4 & 4 & -j4 \\ 0 & -j4 & 6+j4 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 161.245 \angle -82.9^\circ \end{pmatrix}$$

$$\Delta = 2-j4 [4(6+j4) - (-j4 \times -j4)]$$

$$= 240 - j64 = 248.39 \angle -14.93^\circ$$

current thro' 4Ω R is I_2 .

$$\Delta I_2 = \begin{vmatrix} 2-j4 & 0 & 0 \\ j4 & 0 & -j4 \\ 0 & 161.245 \angle -82.9^\circ & 6+j4 \end{vmatrix}$$

$$161.245 \angle -82.9^\circ = 19.93 - j160$$

6

~~16/24~~

$$\Delta I_2 = \begin{vmatrix} 2-j4 & 0 \\ j4 & 0 \\ 0 & 19.93-j160 \end{vmatrix} \begin{vmatrix} 0 \\ -j4 \\ 6+j4 \end{vmatrix}$$

$$\Delta I_2 = 2-j4 \begin{bmatrix} -(-j4 \times (19.93-j160)) \\ -0+0 \end{bmatrix}$$

$$= 2-j4 \begin{bmatrix} -(-j79.72 - 640) \end{bmatrix}$$

$$= 2-j4 \begin{bmatrix} j79.72 + 640 \end{bmatrix}$$

$$= j159.44 + 1280 + 318.88 - j2560$$

$$\Delta I_2 = 1598.88 - j2400.56$$

$$\Delta I_2 = \cancel{2488} - 2884.29 \angle -56.33^\circ$$

By Cramer's Rule

$$I_2 = \frac{\Delta I_2}{D} = \frac{2884.29 \angle -56.33}{248.39 \angle -14.93}$$

$$= 11.612 \angle -56.33 - (-14.93)$$

$$I_{2B} = 11.612 \angle -41.4^\circ \text{ A}$$

$$\cancel{I_2} = \cancel{8.71 \angle -7.68^\circ \text{ A}}$$

$$I_{2B} = 8.71 - j7.68 \text{ A}$$

By Superposition Theorem

⑦

$$I_2 = +I_{2A} - I_{2B}$$

$$= (8.711 - j7.67) - (8.71 - j7.68)$$

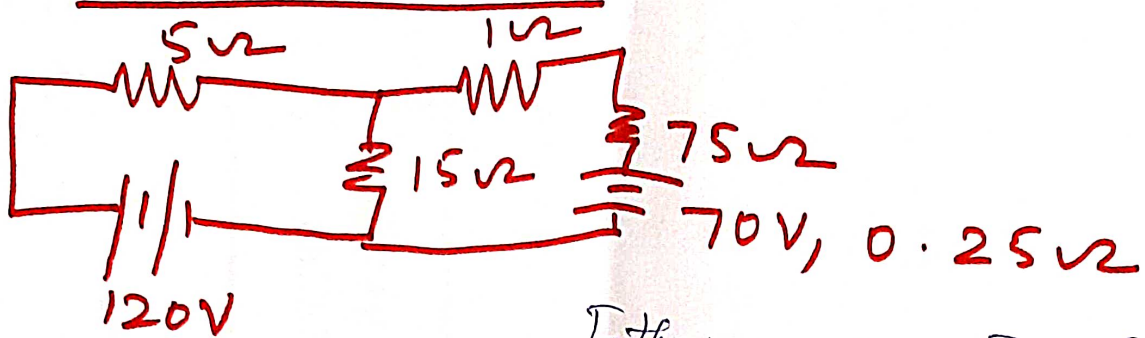
$$= 8.711 - j7.67 - 8.71 + j7.68$$

$$= 0$$

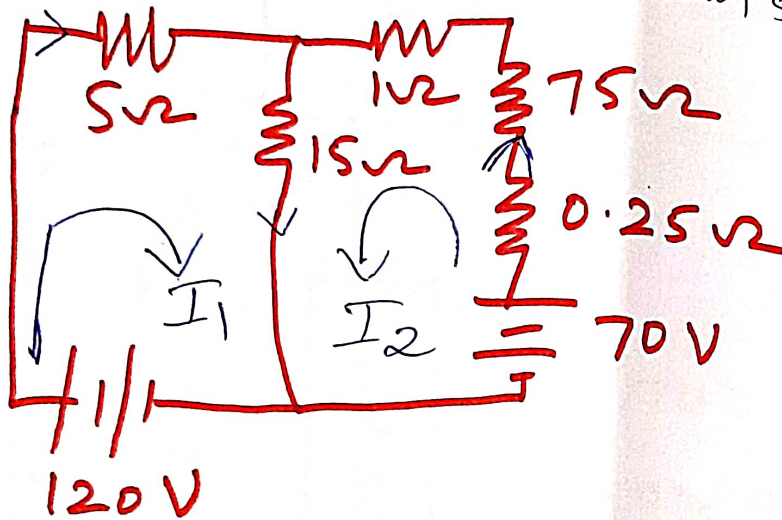
Superposition Theorem

DC circuits

(1)



$$I_{tho, 15\Omega R} = I_1 + I_2$$



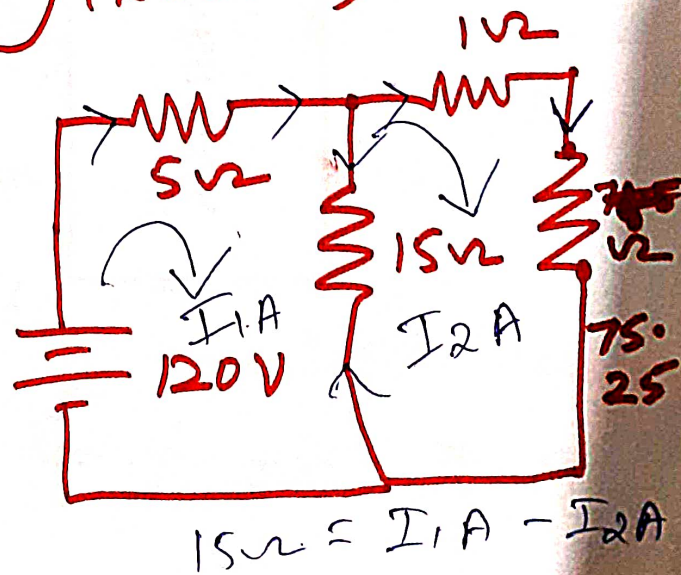
← original circuit given in problem

Step 1 : Draw loop currents in original ~~circuit~~ circuits

As per superposition theorem, use single power supply (source) at a time.

Step 2
only 120V source

Draw loop currents
 $I_1 A, I_2 A$



Use KVL, find $I_1 A$ and $I_2 A$

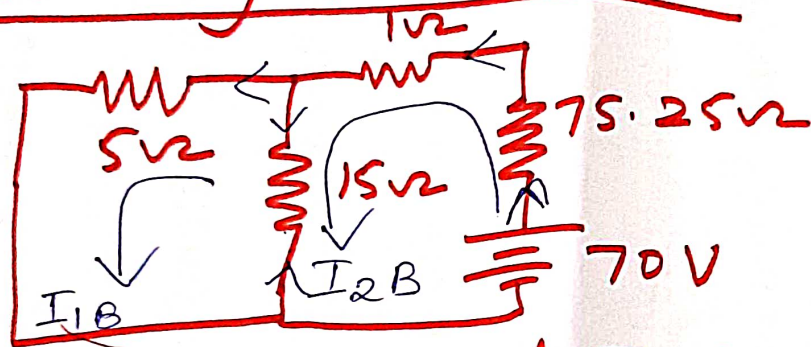
(2)

$$\begin{vmatrix} 20 & -15 \\ -15 & 91.25 \end{vmatrix} \begin{vmatrix} I_1 A \\ I_2 A \end{vmatrix} = \begin{vmatrix} 120 \\ 0 \end{vmatrix}$$

$$I_1 A = 6.84 A \quad I_2 A = 1.125 A$$

step 3

Use only 70V source



Draw loop
currents
 $I_2 A$
 $I_2 B$

Use KVL, find $I_2 A$, $I_2 B$

$$15 \Omega R = I_{1B} - I_{2B}$$

$$\begin{vmatrix} 20 & -15 \\ -15 & 91.25 \end{vmatrix} \begin{vmatrix} I_{1B} \\ I_{2B} \end{vmatrix} = \begin{vmatrix} 0 \\ 70 \end{vmatrix}$$

$$\cancel{I_1 A} = 0.65 A, I_{2B} = 0.875 A$$

I_{1B}

According to Superposition theorem,

$I_1 = I_{1A} + I_{1B}$ for understanding provided see the direction of $I_1 A$ and $I_1 B$ w.r.t I_1

(3)

direction of $I_1 A$ same as $I_1 \Rightarrow +$
direction of $I_1 B$ opposite to $I_1 \Rightarrow (-)$

$$I_1 = +I_{1A} - I_{1B} \Rightarrow \text{actual formula according to the given circuit.}$$

$$I_1 = 6.84 - 0.65 = 6.19 A.$$

$$I_2 = -I_{2A} + I_{2B}$$

direction of I_2 opposite to $I_{2A} \Rightarrow (-ve) \text{ sign}$

direction of I_2 same as $I_{2B} \Rightarrow (+ve) \text{ sign.}$

$$= -(1.125) + 0.875$$

$$= -0.245 A (= 0.245 A)$$

Current in ~~75~~ 75Ω R as per superposition theorem.

$$I_{75\Omega} = -I_{2A} + I_{2B}$$

$$= -(1.125) + 0.875$$

$$= -0.25 A = (0.25 A)$$

Current in 5Ω R as per superposition theorem. (4)

$$\begin{aligned} I_{5\Omega} &= I_{1A} - I_{1B} \\ &= 6.84 - (0.65) \\ &= 6.19 \text{ A} \end{aligned}$$

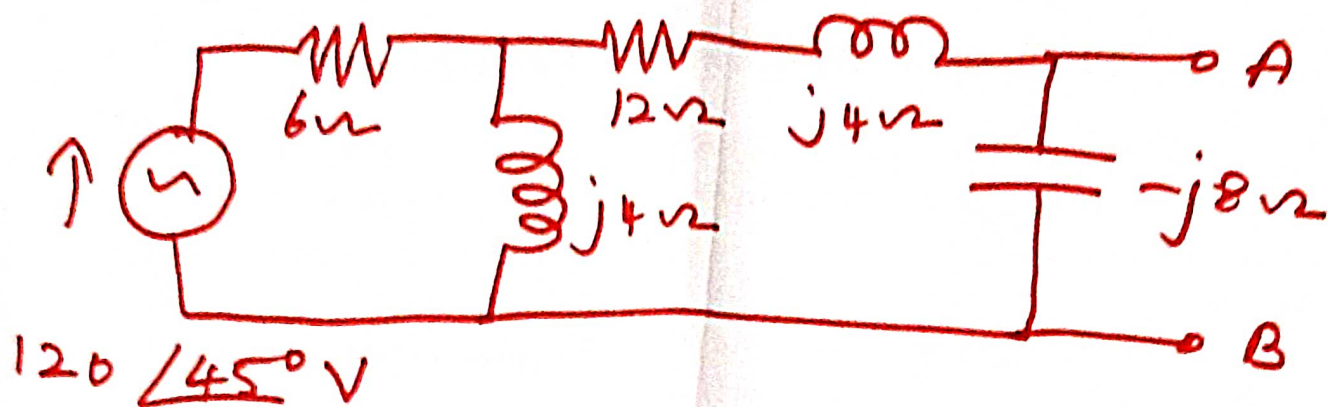
$$\begin{aligned} V_{5\Omega} &= I_{5\Omega} \cdot R \\ &= 6.19 \times 5 = 30.95 \end{aligned}$$

Using Superposition theorem, we can calculate loop currents, current thro' each element (R) and voltage across each element.

Loop current in original circuit is equal to the algebraic sum of loop current when individual voltage source is acting alone.

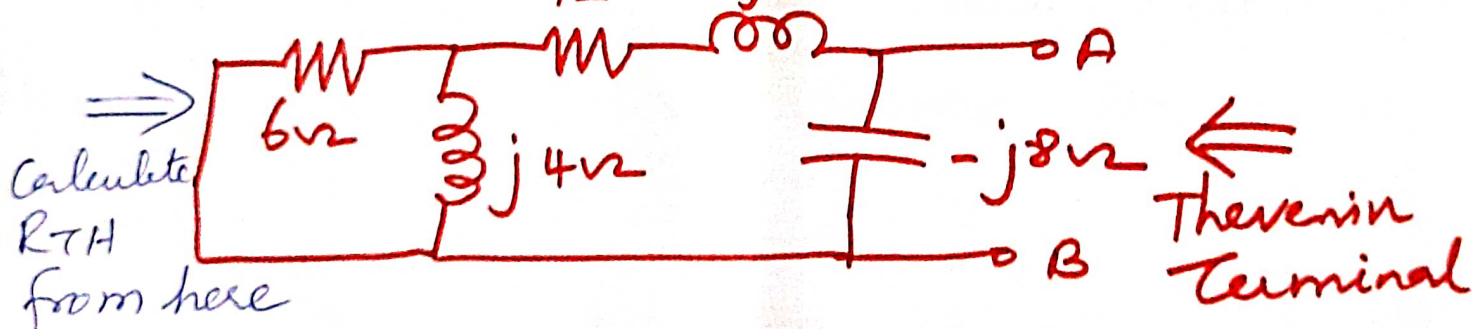
Thevenin Theorem - AC circuits ①

Find Thevenin's equivalent at terminals AB for the circuit shown in figure.



$R_L = 0$, No element connected between A & B.

$R_{TH} \rightarrow \text{step 1} = Z_{TH} = \text{Effective Resistance / Impedance}$



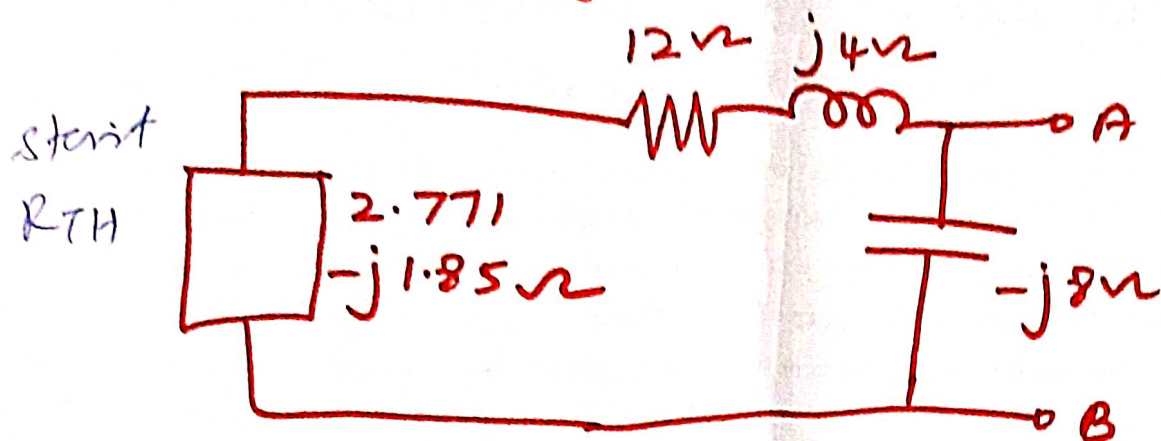
$$6\Omega \parallel j4\Omega$$

$$\frac{6 \times j4}{6 + j4} = \frac{j24}{6 + j4} = \frac{\sqrt{0^2 + 24^2} \angle \tan^{-1}\left(\frac{24}{0}\right)}{\sqrt{6^2 + 4^2}}$$

$$= \frac{24 \angle 0^\circ}{7.211 \angle 33.69^\circ} = 3.33 \angle -33.69^\circ$$

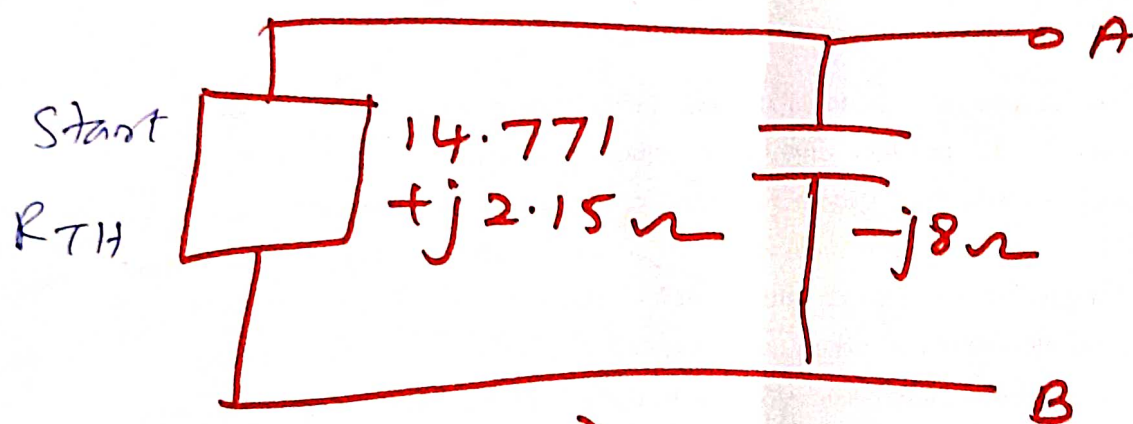
(2)

$$= 3.33 \cos(-33.69) + j 3.33 \sin(-33.69)$$
$$= 2.771 - j1.85$$



2.771 - j1.85 series 12 Ω series j4 Ω

$$2.771 - j1.85 + 12 + j4 = 14.771 + j2.15$$



$$\frac{(14.771 + j2.15)(-j8)}{14.771 + j2.15 - j8} = \frac{-j118.168 + 17.2}{14.771 - j5.85}$$
$$= \frac{17.2 - j118.168}{14.771 - j5.85}$$

$$= \sqrt{17.2^2 + 118.168^2} \tan^{-1} \left(\frac{-118.168}{17.2} \right) \quad (3)$$

$$\sqrt{14.771^2 + 5.85^2} \tan^{-1} \left(\frac{-5.85}{14.771} \right)$$

$$= \frac{119.41 \angle -81.72}{15.89 \angle -21.61} = 7.51 \angle -60.11$$

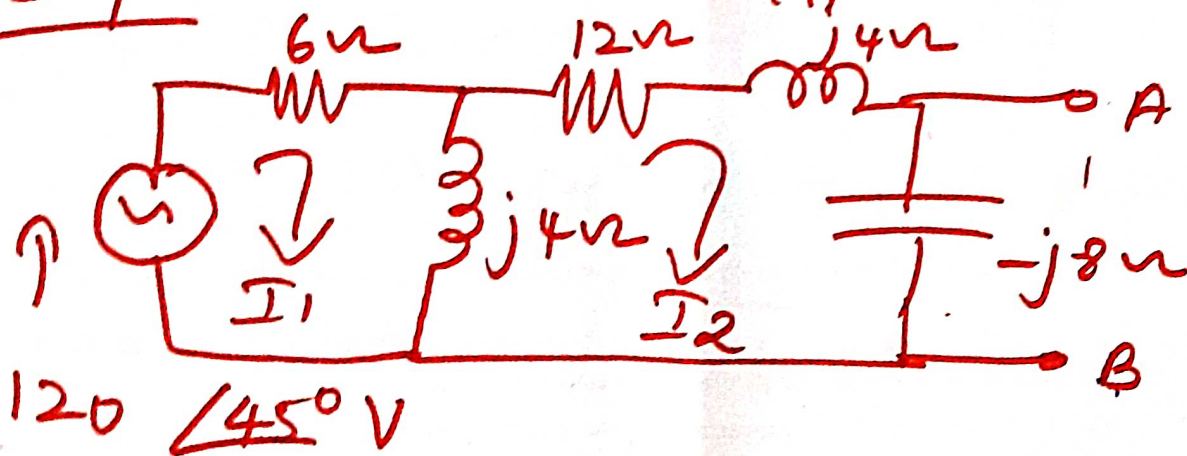
$$= 7.51 \cos(-60.11) + j 7.51 \sin(-60.11)$$

$$= 3.74 - j 6.511 = Z_{TH}$$

$$= \sqrt{3.74^2 + 6.511^2} \tan^{-1} \left(\frac{-6.511}{3.74} \right)$$

$$= 7.51 \angle -60.13 \Omega = Z_{TH} = R_{TH}$$

Step 2: calculate V_{TH}



V_{TH} = Voltage across AB

= Voltage across $-j8 \Omega$

Voltage in parallel path is same

To find voltage across $-j8\Omega$, we need I_2 . (4)

$$|Z| |I| = |V|$$

$$|R| |I| = |V|$$

$$\begin{vmatrix} 6+j4 & -j4 \\ -j4 & 12+j4+j4-j8 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 120 \angle 45^\circ \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} 6+j4 & -j4 \\ -j4 & 12 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 120 \angle 45^\circ \\ 0 \end{vmatrix}$$

$$D = \begin{vmatrix} 6+j4 & -j4 \\ -j4 & 12 \end{vmatrix}$$

$$= (6+j4)(12) - [-j4 \times -j4] \begin{matrix} +j^2 16 \\ -16 \end{matrix}$$

$$= (72+j48) - (-16) = 72+j48+16$$

~~$$= 56+j48 = \sqrt{56^2+48^2} \tan^{-1} \left(\frac{48}{56} \right)$$~~

~~$$= 73.76$$~~

$$= 88+j48 = \sqrt{88^2+48^2} \tan^{-1} \left(\frac{48}{88} \right)$$

$$= 100.24 \angle 28.61^\circ = \Delta$$

$$\Delta I_2 = \begin{vmatrix} 6+j4 & 120 \angle 45^\circ \\ -j4 & 0 \end{vmatrix}$$

$$120 \angle 45^\circ = 120 \cos 45^\circ + j 120 \sin 45^\circ \\ = 84.85 + j 84.85$$

$$\Delta I_2 = \begin{vmatrix} 6+j4 & 84.85 + j 84.85 \\ -j4 & 0 \end{vmatrix}$$

$$= (6+j4)0 - (-j4(84.85 + j 84.85))$$

$$= -j 339.4 - j^2 339.4$$

$$= 339.4 - j 339.4 = \Delta I_2$$

$$= 339.4 \cos(-)$$

$$= \sqrt{339.4^2 + 339.4^2} \tan^{-1} \left(\frac{-339.4}{339.4} \right)$$

$$= 479.98 \angle -45^\circ$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{479.98 \angle -45^\circ}{100.24 \angle 28.61^\circ}$$

$$= 4.788 \angle -73.61^\circ \text{ A} = I_2$$

(6)

$$= 4.788 \cos(-73.61) + j 4.788 \sin(-73.61)$$

$$= 1.35 - j 4.59 \text{ A} = I_2$$

$$V_{across -j8\Omega} = I_2 \times -j8 = V = \frac{I R}{I_2}$$

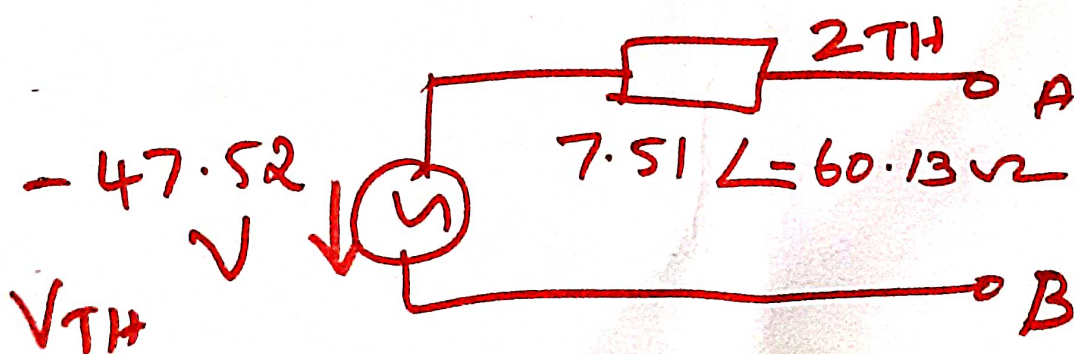
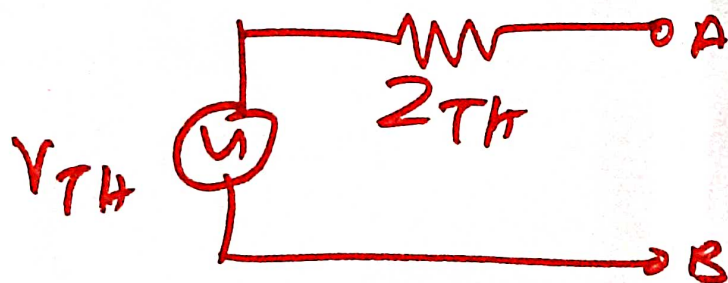
$$= 4.788 \angle -73.61^\circ \times -j8$$

$$= (1.35 - j 4.59)(-j8)$$

$$= -10.8 + j^2 36.72$$

$$V_{TH} = -10.8 - 36.72 = -47.52 \text{ V.}$$

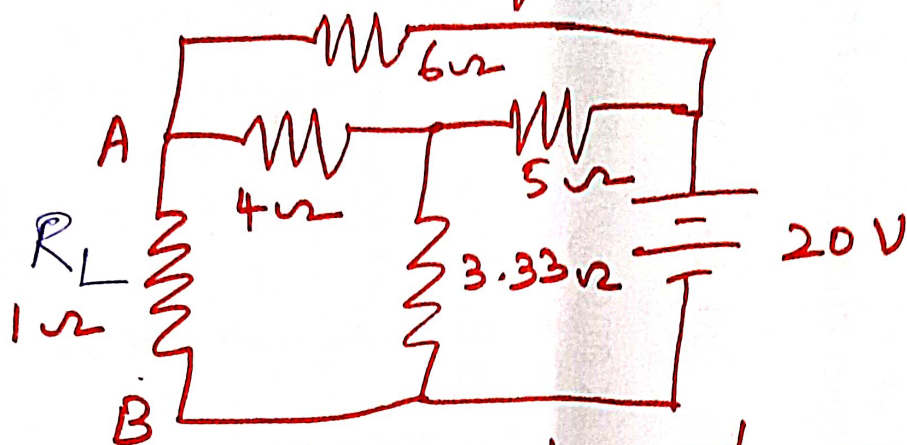
Thevenin equivalent circuit.



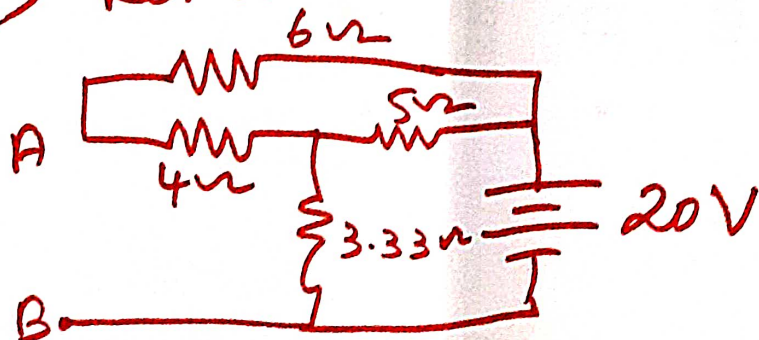
Thevenin's Theorem ^{DC circuits}

(1)

Find the open circuit voltage at AB, Thevenin's Resistance at AB and current thro' the resistance AB as shown in figure. Draw Thevenin's equivalent circuit.

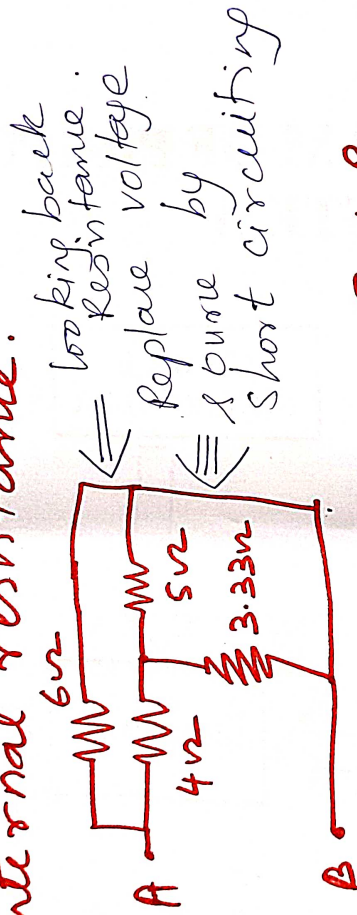


1. Apply Thevenin's Theorem across AB
2. ~~Load~~ Load Resistance $R_L =$
Resistor across AB $= 1\Omega$
3. Calculate Thevenin's Resistance R_{TH}
3.1 $\Rightarrow$ Remove R_L



3.2, Replace voltage source by short circuit, (Replace current source by open circuit) ②

⇓
Replace all sources by their internal resistance.



To the above circuit, find R_{TH}
 $R_{TH} \Rightarrow$ looking back resistance
(total resistance) from the thevenin's terminal AB.
calculate effective (total) resistance starting from opposite side of AB.

3

5Ω, 3.33Ω parallel

$$\frac{5 \times 3.33}{5 + 3.33} = 1.99$$

$$5 + 3.33$$



4Ω series with 1.99Ω

$$4 + 1.99 = 5.99\Omega$$



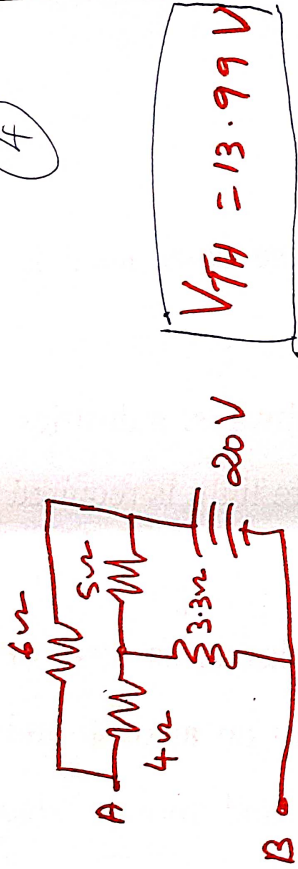
6Ω parallel 5.99Ω

$$\frac{6 \times 5.99}{6 + 5.99} = 2.99\Omega$$

$$R_{TH} = 2.99\Omega$$

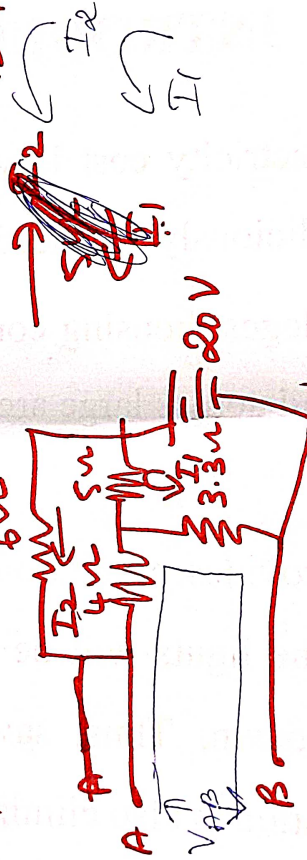
4. Find $V_{TH} \Rightarrow$ Thevenin's Voltage.

4.1, Remove R_L



$V_{TH} \Rightarrow$ voltage across AB

$$V_{TH} = V_{4\Omega} + V_{3.3\Omega} = 13.99V$$



$$\begin{vmatrix} 8.33 & -5 \\ -5 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 20 \\ 0 \end{vmatrix}$$

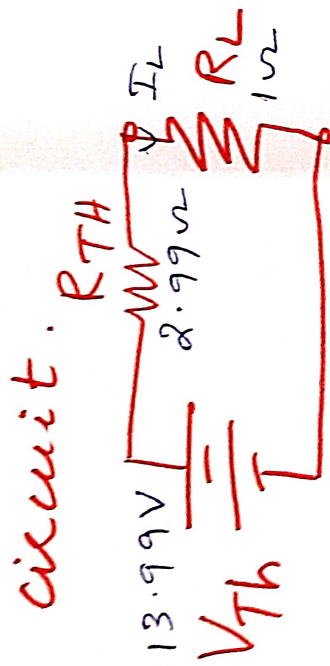
$$I_1 = 3A$$

$$I_2 = 4A$$

$$V_{3.3\Omega} = I_1 \times 3.3 = 9.99$$

$$V_{4\Omega} = I_2 \times 4 = 16$$

5. Draw Thevenin equivalent circuit.



6. Calculate load current I_L

I_L is flowing R_L

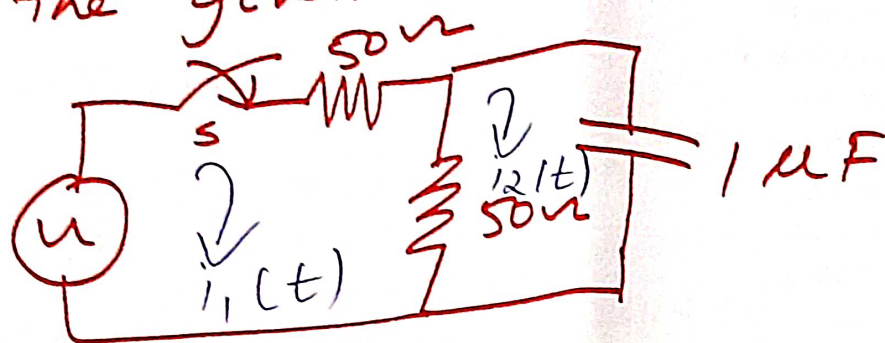
$$I = \frac{V}{R}$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{13.99}{2.99 + 1} = 3.5 A$$

AC Transient - Parallel.

Find Transient currents $i_1(t)$ and $i_2(t)$ for the given circuit ①



100 sin 50t

AC parallel circuit

Two loops $\Rightarrow i_1(t)$ and $i_2(t)$

Loop ①

$$50 i_1(t) + 50(i_1(t) - i_2(t)) = 100 \sin 50t$$

$$50 i_1(t) + 50 i_1(t) - 50 i_2(t) = 100 \sin 50t$$

$$100 i_1(t) - 50 i_2(t) = 100 \sin 50t$$

By Laplace Transform

$$100 I_1(s) - 50 I_2(s) = 100 \cdot \frac{50}{s^2 + 50^2}$$

$$100 I_1(s) - 50 I_2(s) = \frac{5000}{s^2 + 50^2} \quad \text{--- ①}$$

Loop ②
$$sD(i_2(t) - i_1(t)) + \frac{1}{1 \times 10^{-6}} \int i_2(t) dt = 0$$

$$sD i_2(t) - sD i_1(t) + 10^6 \int i_2(t) dt = 0$$

By Laplace Transform

$$-sD I_1(s) + sD I_2(s) + 10^6 \frac{I_2(s)}{s} = 0$$

$$-sD I_1(s) + I_2(s) \left[\frac{sD + 10^6}{s} \right] = 0$$

$$-sD I_1(s) + I_2(s) \left[\frac{sD + 10^6}{s} \right] = 0 \quad \text{--- ②}$$

By matrix, find $I_1(s), I_2(s)$

$$\begin{vmatrix} 100 & -sD \\ -sD & \frac{sD + 10^6}{s} \end{vmatrix} \begin{vmatrix} I_1(s) \\ I_2(s) \end{vmatrix} = \begin{vmatrix} \frac{sD D}{s^2 + sD^2} \\ 0 \end{vmatrix}$$

$$D = \begin{vmatrix} 100 & -sD \\ -sD & \frac{sD + 10^6}{s} \end{vmatrix} = \frac{sD D + 10^6 sD - 2sD D}{s}$$

$$\Delta I_1(s) = \begin{vmatrix} \frac{sD D}{s^2 + sD^2} & 0 \\ -sD & \frac{sD + 10^6}{s} \end{vmatrix} \begin{vmatrix} -sD \\ \frac{sD + 10^6}{s} \end{vmatrix}$$

5.

$$DT(1s) = \frac{5000}{s^2 + 50^2} \left(\frac{50s + 10^6}{s} \right)$$

$$T(1s) = \frac{DT(1s)}{D}$$

$$= \frac{5000}{(s^2 + 50^2)} \left(\frac{50s + 10^6}{s} \right) \times \frac{s}{5000s + 10^8} - 2500s$$

$$= \frac{250000s + (5 \times 10^9)}{(s^2 + 50^2) (2500s + 10^8)}$$

$$\div 2500$$

$$= \frac{100s + 200000}{(s^2 + 50^2) (s + 40000)} = T(1s)$$

$$(s^2 + 50^2) (s + 40000)$$

By Partial Fraction

$$= \frac{As + B}{s^2 + 50^2} + \frac{C}{s + 40000} = T(1s)$$

$$(As + B)(s + 40000) + C(s^2 + 50^2) = 100s + 200000$$

$$As^2 + 4000As + Bs + 4000B + Cs^2 + 2500C = 100s + 200000$$

Grouping similar 's' terms

$$s^2(A+C) + s(4000A+B)$$

$$+ (4000B + 2500C) = 100s + 200000$$

Comparing similar 's' terms in LHS with RHS

$$A+C = 0$$

$$4000A+B = 100$$

$$4000B+2500C = 200000$$

By matrix, find A, B, C

$$\begin{pmatrix} 1 & 0 & 1 \\ 4000 & 1 & 0 \\ 0 & 4000 & 2500 \end{pmatrix} \begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 0 \\ 100 \\ 200000 \end{vmatrix}$$

5

$$A = 1.249 \times 10^{-3}$$

$$B = 50$$

$$C = -1.249 \times 10^{-3}$$

$$I_1(s) = \frac{1.249 \times 10^{-3} s + 50}{s^2 + 50^2} - \frac{1.249 \times 10^{-3}}{s + 40000}$$

$$= 1.249 \times 10^{-3} \left(\frac{s}{s^2 + 50^2} \right) + \left(\frac{50}{s^2 + 50^2} \right)$$

$$- 1.249 \times 10^{-3} \left(\frac{1}{s + 40000} \right)$$

By Inverse Laplace Transform

$$\underline{i_1(t)} = 1.249 \times 10^{-3} \cos 50t + \sin 50t - 1.249 \times 10^{-3} e^{-40000t} A$$

$$\Delta I_2 = \begin{vmatrix} 100 & \frac{5000}{s^2 + 50^2} \\ -50 & 0 \end{vmatrix} = \frac{250000}{s^2 + 50^2}$$

$$I_2(s) = \frac{\Delta I_2}{\Delta} = \frac{250000}{s^2 + 50^2} \times \frac{s}{\underline{2500s + 10^8}} \div 2500$$

6

$$= \frac{100}{s^2 + 50^2} \cdot \frac{s}{s + 40000}$$

$$I_2(s) = \frac{100s}{(s^2 + 50^2)(s + 40000)}$$

By Partial Fraction

$$= \frac{As + B}{s^2 + 50^2} + \frac{C}{s + 40000} = I_2(s)$$

cross multiply, Dr cancel

$$(As + B)(s + 40000) + C(s^2 + 50^2) = 100s$$

$$As^2 + 40000As + Bs + 40000B + Cs^2$$

$$+ 2500C = 100s$$

Grouping similar 's' terms

$$s^2(A + C) + s(40000A + B) +$$

$$(40000B + 2500C) = 100s$$

Comparing 's' terms in LHS and RHS

$$A + C = 0$$

$$40000A + B = 100$$

$$40000B + 2500C = 0$$

By matrix, find A, B, C

⑦

$$\begin{pmatrix} 1 & 0 & 1 \\ 40000 & 1 & 0 \\ 0 & 40000 & 2500 \end{pmatrix} \begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 0 \\ 100 \\ 0 \end{vmatrix}$$

$$A = 2.499 \times 10^{-3}$$

$$B = 1.562 \times 10^{-4}$$

$$C = -2.499 \times 10^{-3}$$

$$I_2(s) = \frac{2.499 \times 10^{-3} s + 1.562 \times 10^{-4}}{s^2 + 50^2}$$

$$= \frac{-2.499 \times 10^{-3}}{s + 40000}$$

$$= 2.499 \times 10^{-3} \left(\frac{s}{s^2 + 50^2} \right) + \frac{1.562 \times 10^{-4}}{50} \left(\frac{50}{s^2 + 50^2} \right)$$

$$- 2.499 \times 10^{-3} \left(\frac{1}{s + 40000} \right)$$

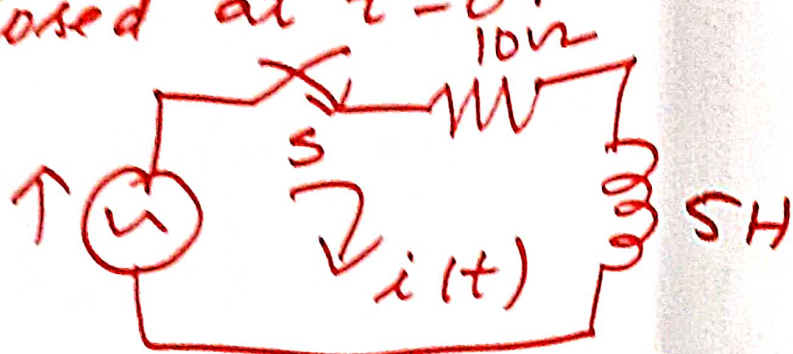
By Inverse Laplace Transform.

$$\underline{i_2(t)} = 2.499 \times 10^{-3} \cos 50t + 3.124 \times 10^{-6} \sin 50t - 2.499 \times 10^{-3} e^{-40000t} A$$

AC Transient Response RL series circuit

(1)

Find transient current for the given circuit when switch is closed at $t=0$.



AC RL series circuit

$50 \sin 25t$ (AC power supply)

By KVL, write differential equation

$$R i(t) + L \frac{di(t)}{dt} = V$$

$$10 i(t) + 5 \frac{di(t)}{dt} = 50 \sin 25t$$

By Laplace Transform

$$\sin at = \frac{a}{s^2 + a^2}$$

$$10 I(s) + 5 [s I(s) - i(0)] = 50 \cdot \frac{25}{s^2 + 25^2}$$

$$10 I(s) + 5s I(s) = \frac{1250}{s^2 + 25^2}$$

$$I(s) [10 + 5s] = \frac{1250}{s^2 + 25^2}$$

$$I(s) = \frac{1250}{(s^2 + 25^2)(s + 10)}$$

$$= \frac{1250}{(s^2 + 25^2)s(s+2)} = \frac{250}{(s^2 + 25^2)(s+2)}$$

By Partial Fraction.

$$(s^2 + a^2) = (s + ja)(s - ja)$$

$$(s^2 + 25^2) = (s + j25)(s - j25)$$

$$I(s) = \frac{250}{(s + j25)(s - j25)(s + 2)}$$

$$= \frac{A}{(s + j25)} + \frac{B}{(s - j25)} + \frac{C}{(s + 2)} = \frac{250}{Ds}$$

cross multiply

$$A(s - j25)(s + 2) + B(s + j25)(s + 2)$$

$$+ C(s + j25)(s - j25) = 250$$

$$\text{put } s = -2$$

$$C(-2+j25)(-2-j25) = 250$$

~~$$4 - j50$$~~

$$C(4 + j50 - j50 - j^2 625) = 250$$

$$C(4 + 625) = 250$$

$$629C = 250$$

$$C = \frac{250}{629} = 0.3975$$

$$\text{put } s = j25$$

$$B(s+j25)(s+2) = 250$$

$$B[(j25+j25)(j25+2)] = 250$$

$$B[j50(j25+2)] = 250$$

$$B[j^2 1250 + j1000] = 250$$

$$B[-1250 + j1000] = 250$$

$$B = \frac{250}{-1250 + j1000}$$

(4)

$$A \text{ put } s = -j25$$

$$A(s - j25)(s + 2) = 250$$

$$A(-j25 - j25)(-j25 + 2) = 250$$

$$A(-j50)(-j25 + 2) = 250$$

$$A(j^2 1250 - j100) = 250$$

$$A(-1250 - j100) = 250$$

$$A = \frac{250}{-1250 - j100}$$

$$-1250 - j100$$

Co. eff of A and B is imaginary
so do partial fraction in a
different method.

5

$$I(s) = \frac{250}{(s^2 + 25^2)(s+2)}$$

we have $s^2 + a^2$ in Denominator
write as $\frac{As+B}{s^2+a^2}$

$$I(s) = \frac{As+B}{(s^2+25^2)} + \frac{C}{(s+2)} = \frac{250}{(s^2+25^2)(s+2)}$$

(By Partial Fraction)

cross multiply

$$(As+B)(s+2) + C(s^2+25^2) = 250$$

Common Dr get cancelled out

$$As^2 + 2As + Bs + 2B + Cs^2 + C25^2 = 250$$

Group like terms of s

$$s^2[A+C] + s[2A+B] + [2B+625C]$$

Comparing s co. eff $\overset{=250}{}$ in RHS and LHS

6

$$A + C = 0$$

$$2A + B = 0$$

$$2B + 625C = 250$$

Write the above three eqn in matrix form to find the value of A, B, C

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 625 \end{vmatrix} \begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 250 \end{vmatrix}$$

$$A = -0.4$$

$$B = 0.79$$

$$C = 0.4$$

co-eff of A, B, C is a real number and not imaginary number.

$$I(s) = \frac{-0.4s + 0.79}{s^2 + 25^2} + \frac{0.4}{s + 2}$$

$$= \frac{-0.4s}{s^2 + 25^2} + \frac{0.79}{s^2 + 25^2} + \frac{0.4}{s + 2}$$

By Inverse Laplace Transform ⑦
 $i(t)$.

$$\frac{s}{s^2 + a^2} = \cos at$$

$$\frac{a}{s^2 + a^2} = \sin at$$

$$\frac{1}{s + a} = e^{-at}$$

$$I(s) = \frac{-0.4s}{s^2 + 25^2} + \frac{25}{25} \left(\frac{0.79}{s^2 + 25^2} \right) + \frac{0.4}{s + 2}$$

arranging to apply formula

$$= -0.4 \left(\frac{s}{s^2 + 25^2} \right) + \frac{0.79}{25} \left(\frac{25}{s^2 + 25^2} \right) + 0.4 \left(\frac{1}{s + 2} \right)$$

$$i(t) = -0.4 \cos 25t + 0.0316 \sin 25t + 0.4 e^{-2t} \text{ A.}$$

(8)

DC Transient Response from a
DC circuit

DC power supply
whole numbers for power
supply eg. 10, 20, 30

AC Transient Response

AC circuit

AC power supply

Number along angle or some
trigonometric functions.

eg. $10 \angle 0^\circ \text{ V}$

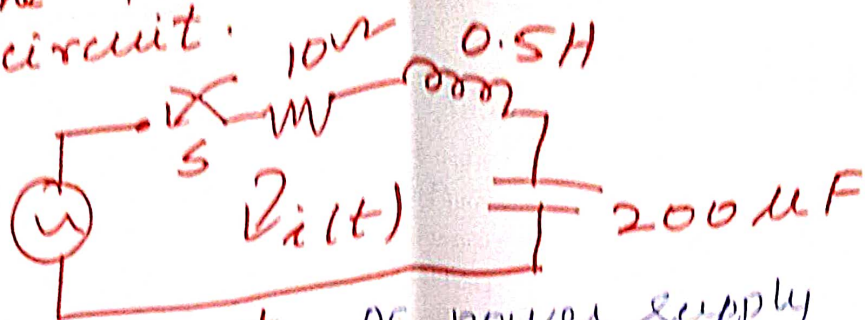
$25 \angle 68^\circ \text{ V}$

$100 \sin \omega t$, $1000 \sin 500t$

$15e^{-10t}$

AC - Transient - RLC series circuit

Determine Transient current eqn for the given circuit. (1)



150 sin 200t - AC power supply

AC RLC series circuit.

By KVL, differential equation

$$10 i(t) + 0.5 \frac{di(t)}{dt} + \frac{1}{200 \times 10^{-6}} \int i(t) dt$$

$$= 150 \sin 200t$$

By Laplace Transform

$$10 I(s) + 0.5 [s I(s) - i(0)] + 5000 \cdot \frac{I(s)}{s}$$

$$= \frac{150 \cdot 200}{s^2 + 200^2}$$

$$10 I(s) + 0.5 s I(s) + 5000 \frac{I(s)}{s} = \frac{30000}{s^2 + 200^2}$$

$$I(s) \left[10 + 0.5 s + \frac{5000}{s} \right] = \frac{30000}{s^2 + 200^2}$$

$$I(s) \left[\frac{10s + 0.5 s^2 + 5000}{s} \right] = \frac{30000}{s^2 + 200^2}$$

$$I(s) = \frac{30000}{s^2 + 200^2} \times \frac{s}{0.5s^2 + 10s + 5000} \quad (2)$$

By Partial Fraction

$$\frac{As + B}{s^2 + 200^2} + \frac{Cs + D}{0.5s^2 + 10s + 5000} = I(s)$$

$$\left\{ (As + B)(0.5s^2 + 10s + 5000) \right\} + \left\{ (Cs + D)(s^2 + 200^2) \right\} = 30000s$$

$$0.5As^3 + 10As^2 + 5000As + 0.5Bs^2 + 10Bs + 5000B + Cs^3 + 40000Cs + Ds^2 + 40000D = 30000s$$

$$s^3(0.5A + C) + s^2(10A + 0.5B + D)$$

$$+ s(5000A + 10B + 40000C) +$$

$$(5000B + 40000D) = 30000s$$

$$0.5A + C = 0$$

$$10A + 0.5B + D = 0$$

$$5000A + 10B + 40000C = 30000$$

$$5000B + 40000D = 0$$

By matrix find value of
A, B, C, D.

Substitute in $I(s)$, simplify it.

By inverse Laplace Transform
find $i(t)$.

Damping conditions

①

Natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$ rad/sec

Damping ratio $\delta = \frac{R}{2\sqrt{L/C}}$ (no unit)

damped frequency $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$ rad/sec

$\omega_d = \omega_n \sqrt{1 - \delta^2}$ rad/sec

Over damped

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

Critically damped

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

(2)

Oscillatory in nature

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

$\delta = 1 \Rightarrow$ critical damping

$\delta < 1 \Rightarrow$ over damping

$\delta > 1 \Rightarrow$ under damping

Natural Response / Free Response

When a circuit containing storage elements which are independent of sources, the response depends upon the nature of the circuit and is called as Natural Response or

Free Response

Forced Response

When a source acting on a circuit, the response depends on the nature of such source (external). The behaviour is determined by the application of external energy is called as forced response.

Coupled circuits

Self induced emf, e

Emf induced in the coil due to change in its own flux linking with it.

Self inductance, L

It is the property of a coil by which it opposes the change in the value of current or flux through it due to production of self induced emf.

Mutually induced emf

The emf induced ^{with} in the coil due to the change in the flux linking ^{with} it, the flux being produced by the current in another coil.

Mutual Inductance (M)

The ability of the first coil to produce an emf in the second coil placed near it, due to change of current or flux in the first coil.

Coupled circuits

If the transfer of energy occurs from one circuit to another through mutual induction, then the two circuits are said to be coupled.

Dot Rule for coupled circuits

If the currents in two mutually coupled coils enter the dotted end or leaves the dotted ends, M terms will have the same sign as L terms.

If the current in one coil enters the dotted end and in the other the current leaves the dotted end, M terms will have the opposite sign as L term.

Co. efficient of coupling

It is defined as the fraction of the magnetic flux produced by the current in one coil that link the other coil.

$$\text{Self induced emf, } e = N \frac{d\phi}{dt} = L \frac{di}{dt}$$

$$\text{Self inductance, } L = \frac{N\phi}{i}$$

$$\text{Co. eff of coupling } K = \frac{M}{\sqrt{L_1 L_2}}$$

$$X_m = K \sqrt{X_1 X_2}$$

$$K = \frac{\phi_{12}}{\phi_1} \left(\frac{\text{flux of I II}}{\text{flux of I coil}} \right)$$

Two coupled coils with self inductances L_1 and L_2 and mutual inductance M have 100 turns and 500 turns respectively. A current of 1A is passed through coil 1 due to which a flux ϕ is set up. The portion of ϕ , that links with coil 1 alone is 1mwb and that link both coil 1 and 2 is 0.3mwb. Find L_1 , L_2 , M and K .

$$\text{Total flux } \phi_1 = \phi_{11} + \phi_{12}$$

$$= 1 \times 10^{-3} + 0.3 \times 10^{-3} = 1.3 \times 10^{-3} \text{ wb} = 1.3 \text{ mwb}$$

$$L_1 = \frac{N_1 \phi_1}{i_1} = \frac{100 \times 1.3 \times 10^{-3}}{1} = 0.13 \text{ H}$$

$$M = \frac{N_2 \phi_{12}}{i_1} = \frac{500 \times 0.3 \times 10^{-3}}{1} = 0.15 \text{ H}$$

$$K = \frac{\phi_{12}}{\phi_1} = \frac{0.3 \times 10^{-3}}{1.3 \times 10^{-3}} = 0.231$$

$$\sqrt{L_1 L_2} = \frac{M}{K} = \frac{0.15}{0.231} = 0.65 \text{ H}$$

$$L_2 = \frac{0.65^2}{0.13} = 3.25 \text{ H}$$

Two coupled coils with self inductances L_1 and L_2 have 500 turns and 1000 turns respectively. The co. eff of coupling between them is 0.8. The portion of flux that links both the coils when a current of 5A flows through coil 1 is 0.9wb. Find values of L_1 and L_2 .

$$\text{Self inductance of coil 1, } L_1 = \frac{N_1 \phi_1}{i_1}$$

$$K = \frac{\phi_{12}}{\phi_1} \Rightarrow \phi_1 = \frac{\phi_{12}}{K} = \frac{0.9}{0.8} = 1.125 \text{ wb}$$

$$L_1 = \frac{500 \times 1.125}{5} = 112.5 \text{ H}$$

$$M = \frac{N_2 \phi_{12}}{i_1} = \frac{1000 \times 0.9}{5} = 180 \text{ H}$$

$$L_2 = \frac{M^2}{K^2 L_1} = 450 \text{ H}$$

Two coupled coils with self inductances $L_1 = 0.04 \text{ H}$ and $L_2 = 0.16 \text{ H}$ and co. eff of coupling $K = 0.6$ has 800 turns in coil 2. The current in coil 1 is $i_1 = 10 \sin 500t$. Find the voltage induced in coil 2 and maximum flux set up by coil 1.

$$L_1 = 0.04 \text{ H}, L_2 = 0.16 \text{ H}, K = 0.6, N_2 = 800$$

$$i_1 = 10 \sin 500t$$

$$M = K \sqrt{L_1 L_2} = 0.6 \sqrt{0.04 \times 0.16} = 0.048 \text{ H}$$

$$e_2 = M \frac{di_1}{dt} = 0.048 \frac{d(10 \sin 500t)}{dt} = 240 \cos 500t$$

$$e_2 = N_2 \frac{d\phi_{12}}{dt} \Rightarrow \phi_{12} = \int \frac{e_2}{N_2} dt = \int \frac{240}{800} \cos 500t dt$$

$$= 6 \times 10^{-4} \sin 500t \text{ wb}$$

$$\phi_1 = \frac{\phi_{12}}{K} = \frac{6 \times 10^{-4} \sin 500t}{0.6} = 10^{-3} \sin 500t \text{ wb}$$

$$\phi_1 = 10^{-3} \text{ wb} = 1 \text{ mwb}$$

Two coupled coils have self inductances $L_1 = 2\text{ H}$, $L_2 = 6\text{ H}$. The co-eff of coupling between them is 0.5. If a current $i_1 = 4 \sin(40t - 30^\circ)\text{ A}$ flows through coil 1 and $i_2 = 2 \sin(40t - 30^\circ)\text{ A}$ flows through coil 2. Find the voltages across coil 1 and coil 2, if the mutually induced emf opposes the self induced emf.

$$L_1 = 2\text{ H}, L_2 = 6\text{ H}, K = 0.5, i_1 = 4 \sin(40t - 30^\circ) \\ i_2 = 2 \sin(40t - 30^\circ).$$

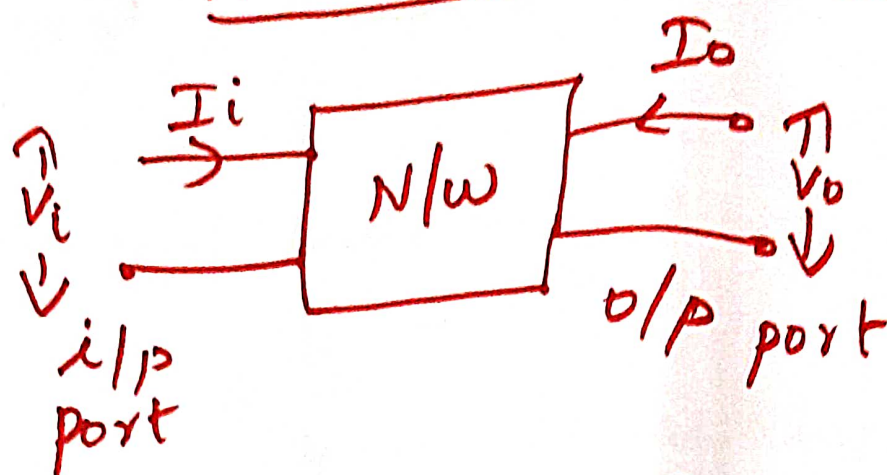
$$M = K \sqrt{L_1 L_2} = 0.5 \sqrt{2 \times 6} = 1.732\text{ H}$$

$$\begin{aligned} \text{Voltage across coil 1, } V_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ &= 2 \frac{d}{dt} (4 \sin(40t - 30^\circ)) - 1.732 \frac{d}{dt} (2 \sin(40t - 30^\circ)) \\ &= 181.44 \cos(40t - 30^\circ)\text{ V}. \end{aligned}$$

$$\begin{aligned} \text{Voltage across coil 2, } V_2 &= L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \\ &= 6 \frac{d}{dt} (2 \sin(40t - 30^\circ)) - 1.732 \frac{d}{dt} (4 \sin(40t - 30^\circ)) \\ &= 202.88 \cos(40t - 30^\circ)\text{ V}. \end{aligned}$$

Network Functions

(1)



Network function: Ratio of response (o/p) to excitation (I/p)

port: A pair of terminals at which an electrical signal enters or leaves a N/w.

port $\Rightarrow$ terminals

Network Functions:

1. Impedance Functions

$$\frac{V_i}{I_i}, \frac{V_o}{I_o}, \frac{V_i}{I_o}, \frac{V_o}{I_i}$$

2. Admittance Functions

$$\frac{I_i}{V_i}, \frac{I_o}{V_o}, \frac{I_i}{V_o}, \frac{I_o}{V_i}$$

$$\text{Voltage Gain} = \frac{V_o}{V_i}$$

$$\text{Gain} = \frac{\text{o/p}}{\text{i/p}} \quad (2)$$

$$\text{Current Gain} = \frac{I_o}{I_i}$$

Driving Point Functions

1. Driving point impedance

$$\frac{V_i}{I_i}, \frac{V_o}{I_o}$$

2. Driving Point Admittance

$$\frac{I_i}{V_i}, \frac{I_o}{V_o}$$

calculated either at i/p port or at o/p port only.

also called as operational point functions or as operational functions

Transfer Function

1. Transfer / Transform Impedance

$$\frac{V_o}{I_i}, \frac{V_i}{I_o}$$

2. Transfer / Transform Admittance

$$\frac{I_i}{V_o}, \frac{I_o}{V_i}$$

3. Voltage Transform / Transfer Ratio

$$\frac{V_o}{V_i}$$

4. Current Transform / Transfer Ratio

$$\frac{I_o}{I_i}$$

Poles and zeros of a N/w function

$$N(s) = \frac{A(s)}{B(s)} = \frac{N_r}{D_r}$$

(4)

$$N(s) = \frac{(s+1)^2 (s+5)}{(s+2) (s+3+j2) (s+3-j2)}$$

Roots of N_r polynomial $\Rightarrow$ zeros
Roots of D_r polynomial $\Rightarrow$ poles.

Zeros: $s = -1, s = -1, s = -5$

Poles: $s = -2, s = -3+j2, s = -3-j2$

$$N(s) = \frac{K (s-z_1) (s-z_2) \dots (s-z_m)}{(s-p_1) (s-p_2) \dots (s-p_N)}$$

$M \rightarrow$ no. of zeros

$N \rightarrow$ no. of poles

$K \rightarrow$ scale factor.

If $N > M$, there are $(N-M)$ zeros at infinity

If $M > N$, there are $(M - N)$ poles at infinity.

Using zeros and poles, $N(s)$ represented as

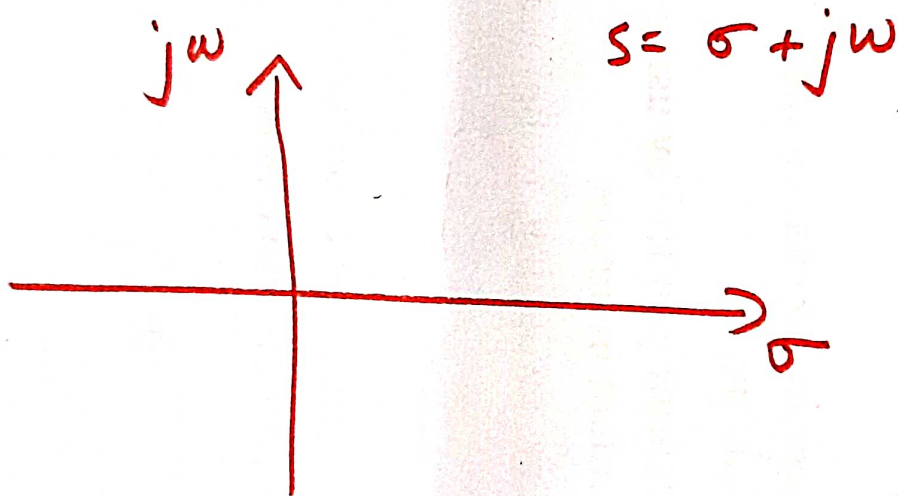
$$N(s) = K(s+1)(s+1)(s+5)$$

$$(s+2)(s+3-j2)(s+3+j2)$$

(5)

Stability of a N/w function

All poles and zeros must lie in left hand side of s -plane.

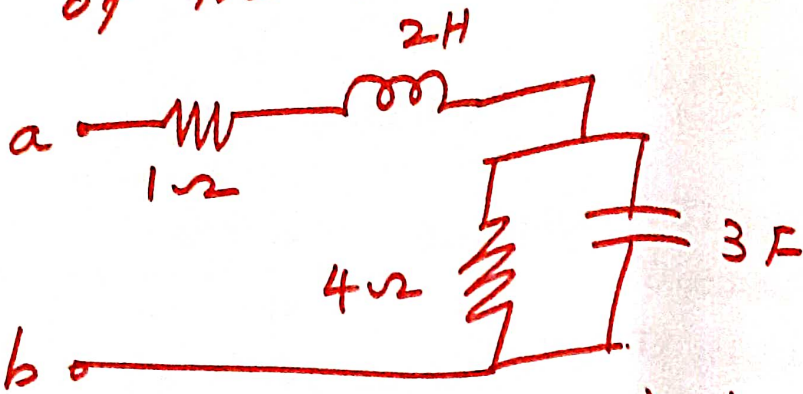


s -plane

$\sigma \rightarrow$ real part

$j\omega \rightarrow$ imaginary part.
 $\omega \rightarrow$

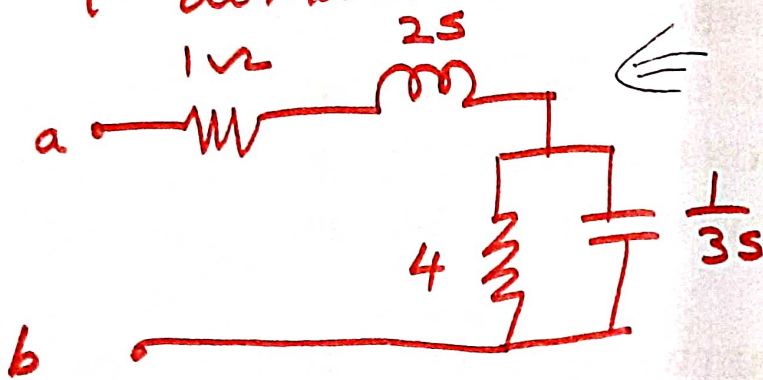
Find the driving point impedance of the network shown in figure.



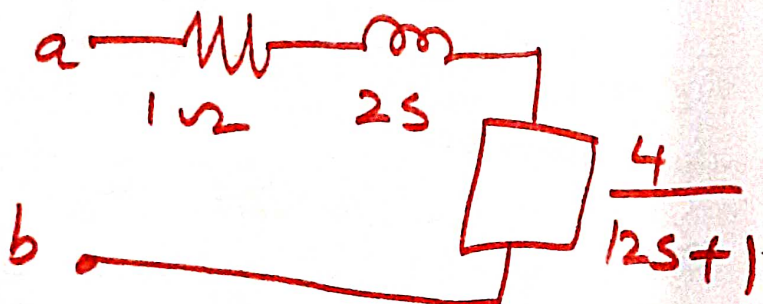
(6)

Given circuit is in 't' domain.

Transform the given circuit into 's' domain



$$\begin{aligned}
 Z(s) &= 1 \parallel \frac{1}{3s} = \frac{4 \times \frac{1}{3s}}{4 + \frac{1}{3s}} = \frac{\frac{4}{3s}}{\frac{12s + 1}{3s}} \\
 &= \frac{4}{3s} \times \frac{3s}{12s + 1} = \frac{4}{12s + 1}
 \end{aligned}$$



All elements in series

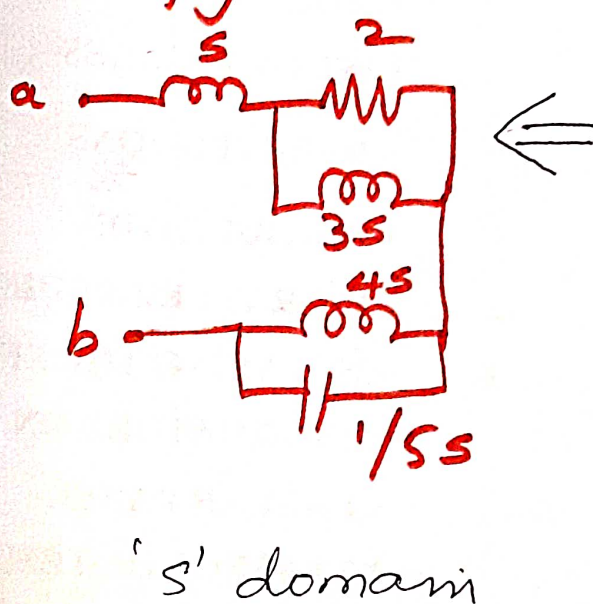
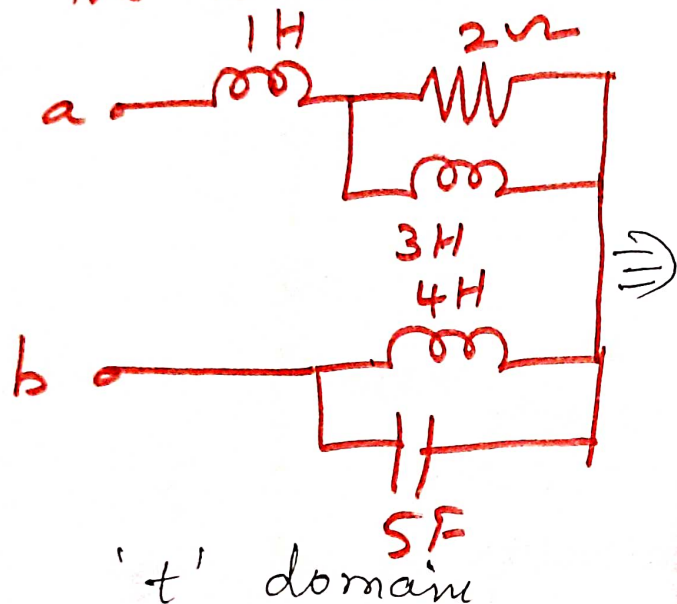
$$= 1 + 2s + \frac{4}{12s+1} = \frac{12s+1}{12s+1} + \frac{24s^2+2s+4}{12s+1}$$

$$= \frac{24s^2 + 14s + 5}{12s+1} = 2(s) = \text{driving point impedance.}$$

(or) Operational impedance.

(or) operational point impedance.

Find the driving point admittance of the network shown in figure.



$$Y(s) = \frac{1}{2(s)}$$

calculate $2(s)$ and find $Y(s)$

$$\left(\frac{2 \parallel 3s}{2 + 3s} \right) + \left(\frac{4s \parallel \frac{1}{5s}}{4s + \frac{1}{5s}} \right)$$

$$\left(\frac{6s}{2+3s} \right) + \left(\frac{\frac{4}{5}}{\frac{20s^2+1}{5s}} \right)$$

8

$$\frac{6s}{2+3s} + \left(\frac{4}{5} \times \frac{5s}{20s^2+1} \right)$$

$$\left(\frac{6s}{2+3s} + \frac{4s}{20s^2+1} \right) \text{ series with 's'}$$

$$\frac{6s}{2+3s} + \frac{4s}{20s^2+1} + 8$$

$$= \frac{6s(20s^2+1) + 4s(2+3s) + 8(2+3s)(20s^2+1)}{(2+3s)(20s^2+1)}$$

$$= \frac{120s^3 + 6s + 8s + 12s^2 + s(40s^2 + 2 + 60s^3 + 3s)}{40s^2 + 2 + 60s^3 + 3s}$$

$$40s^2 + 2 + 60s^3 + 3s$$

$$120s^3 + 14s + 12s^2 + 40s^3 + 2s + 60s^4 + 3s^2$$

$$40s^2 + 60s^3 + 3s + 2$$

$$60s^4 + 160s^3 + 15s^2 + 16s$$

(9)

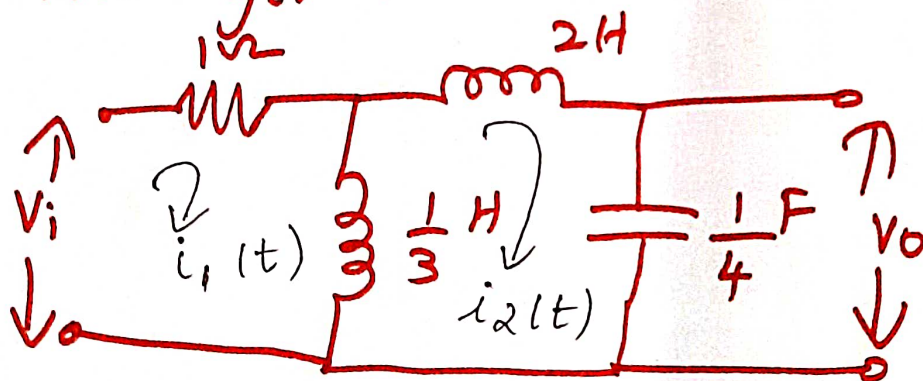
$$\frac{60s^4 + 160s^3 + 15s^2 + 16s}{60s^3 + 40s^2 + 3s + 2} = 2(s)$$

$$Y(s) = \frac{1}{2(s)}$$

$$Y(s) = \frac{60s^3 + 40s^2 + 3s + 2}{60s^4 + 160s^3 + 15s^2 + 16s}$$

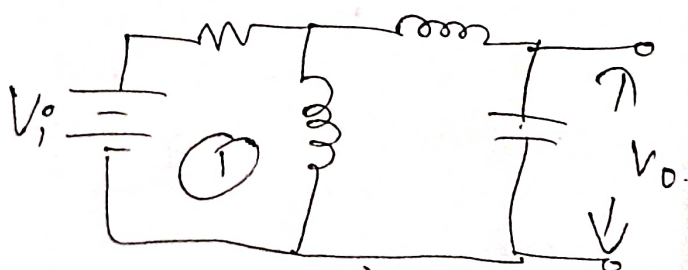
$$60s^4 + 160s^3 + 15s^2 + 16s$$

Find the transfer function $\frac{V_o(s)}{V_i(s)}$ for the given circuit



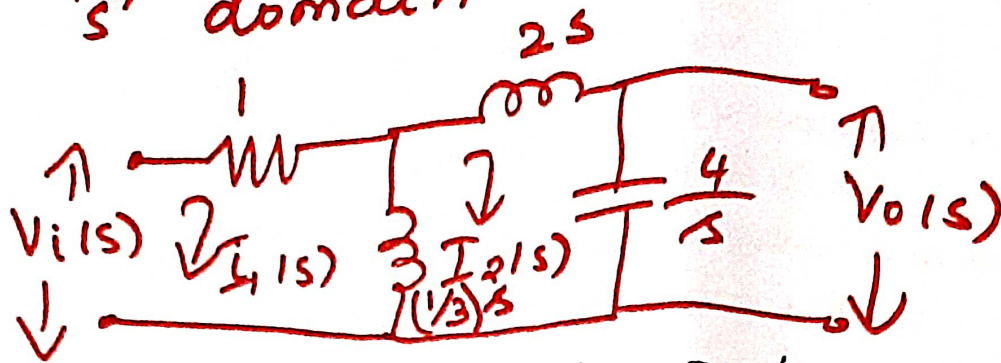
as we know $\frac{1}{4} F$
we will get
o/p " V_o "

't' domain



V_i is the i/p voltage; $\therefore$ closed loop (1)

Transform the given circuit into 's' domain



(16)

$$C = \frac{1}{Cs} = \frac{1}{\frac{1}{4}s} = \frac{4}{s}$$

$$L = \frac{Ls}{3} = \frac{1}{3}s = \frac{s}{3}$$

's' domain circuit

By mesh analysis

$$\begin{vmatrix} 1 + \frac{s}{3} & -\frac{s}{3} \\ -\frac{s}{3} & 2s + \frac{s}{3} + \frac{4}{s} \end{vmatrix} \begin{vmatrix} I_1(s) \\ I_2(s) \end{vmatrix} = \begin{vmatrix} V_i(s) \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} \frac{3+s}{3} & -\frac{s}{3} \\ -\frac{s}{3} & \frac{6s^2 + s^2 + 12}{3s} \end{vmatrix} \begin{vmatrix} I_1(s) \\ I_2(s) \end{vmatrix} = \begin{vmatrix} V_i(s) \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} \frac{3+s}{3} & -s/3 \\ -\frac{s}{3} & \frac{7s^2 + 12}{3s} \end{vmatrix} \begin{vmatrix} I_1(s) \\ I_2(s) \end{vmatrix} = \begin{vmatrix} V_i(s) \\ 0 \end{vmatrix}$$

$$\begin{aligned}
 \Delta &= \frac{3+s}{3} \left[\frac{7s^2+12}{3s} \right] - \left[\frac{-s}{3} \times \frac{-s}{3} \right] \\
 &= \frac{21s^2 + 36 + 7s^3 + 12s}{9s} - \frac{s^2}{9} \quad (11) \\
 &= \frac{189s^2 + 324 + 63s^3 + 108s - 9s^3}{81s} \\
 &= \frac{54s^3 + 189s^2 + 108s + 324}{81s} = \Delta.
 \end{aligned}$$

$$V_o(s) = I_2(s) \cdot \left(\frac{4}{s} \right)$$

$$\Delta I_2(s) = \begin{vmatrix} \frac{3+s}{3} & V_i(s) \\ \frac{-s}{3} & 0 \end{vmatrix}$$

$$\Delta I_2(s) = V_i(s) \cdot \frac{s}{3}$$

$$I_2(s) = \frac{\Delta I_2(s)}{\Delta}$$

$$= V_i(s) \cdot \frac{s}{3} \times \frac{81s}{54s^3 + 189s^2 + 108s + 324}$$

$$V_o(s) = I_2(s) \cdot \frac{4}{s}$$

$$= V_i(s) \cdot \frac{s}{3} \times \frac{81s}{54s^3 + 189s^2 + 108s + 324} \times \frac{4}{s}$$

(12)

$$V_o(s) = V_i(s) \cdot \frac{324s}{162s^3 + 567s^2 + 324s + 972}$$

Transfer function $\frac{V_o(s)}{V_i(s)}$

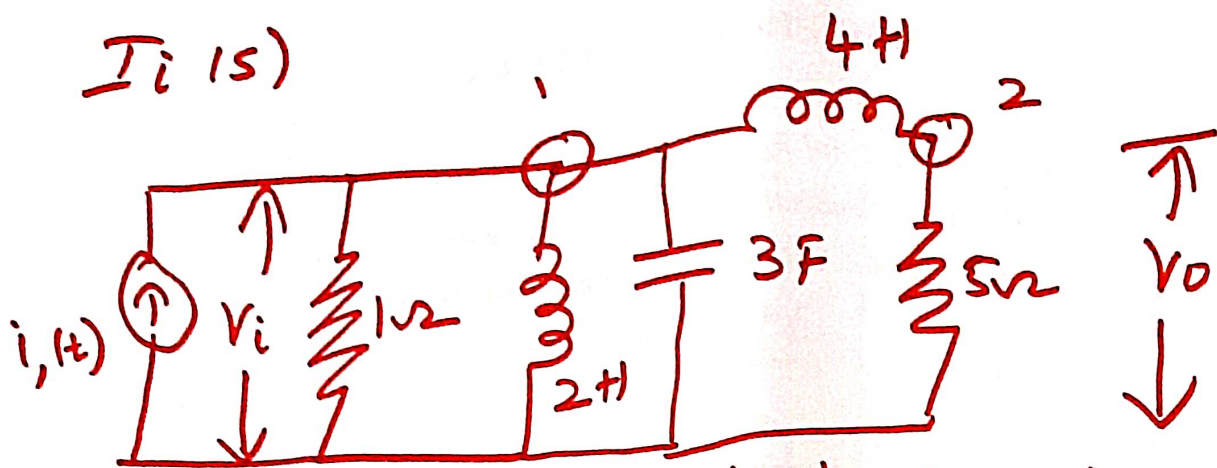
$$\frac{V_o(s)}{V_i(s)} = \frac{324s}{162s^3 + 567s^2 + 324s + 972}$$

$\div 81$ (common term)

$$\frac{V_o(s)}{V_i(s)} = \frac{4s}{2s^3 + 7s^2 + 4s + 12}$$

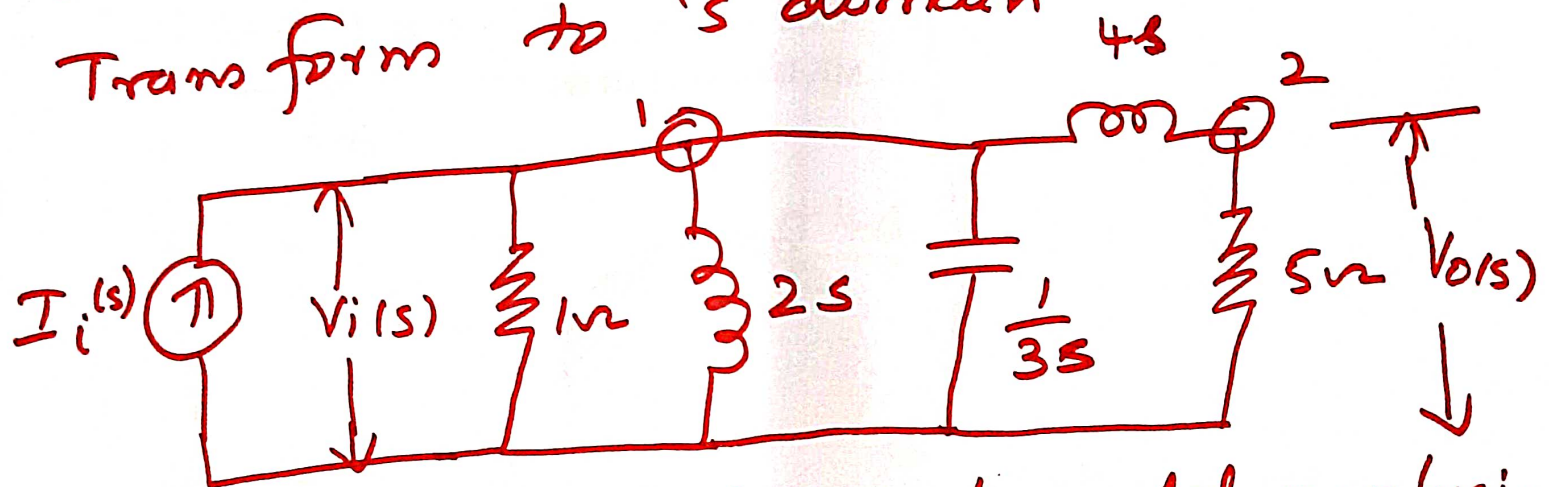
In the given circuit find $\underline{V_o(s)}$ and $\underline{I_o(s)}$

13



Given circuit in 't' domain

Transform to 's' domain



Since node is given, do nodal analysis
Nodal analysis $\Rightarrow$ KCL

$$\begin{vmatrix} \frac{1}{2s} + \frac{1}{1} + \frac{1}{1/3s} + \frac{1}{4s} & -\frac{1}{4s} \\ -\frac{1}{4s} & \frac{1}{5} + \frac{1}{4s} \end{vmatrix} \begin{vmatrix} V_o(s) \\ V_o(s) \end{vmatrix}$$

$$= \begin{vmatrix} I(1s) \\ 0 \end{vmatrix}$$

14

$$= \begin{vmatrix} \frac{12s^2 + 48s + 3}{4s} & -\frac{1}{4s} \\ -\frac{1}{4s} & \frac{5 + 48}{20s} \end{vmatrix} \begin{vmatrix} V(1s) \\ V_0(1s) \end{vmatrix} = \begin{vmatrix} I(1s) \\ 0 \end{vmatrix}$$

$$D = \begin{vmatrix} \frac{12s^2 + 48s + 3}{4s} & -\frac{1}{4s} \\ -\frac{1}{4s} & \frac{5 + 48}{20s} \end{vmatrix}$$

$$= \frac{60s^2 + 208 + 15 + 48s^3 + 16s^2 + 12}{80s^2} - \frac{1}{16s^2}$$

$$= \frac{16(60s^2 + 208 + 15 + 48s^3 + 16s^2 + 12) - 80}{1280s^2}$$

$$= \frac{768s^3 + 1216s^2 + 3208 + 432 - 80}{1280s^2}$$

$$\frac{768s^3 + 1216s^2 + 320s + 352}{1280s^2} = \Delta \quad (15)$$

$$\Delta V_{i1s} = \left| \begin{array}{cc} I_{i1s} & -\frac{1}{4s} \\ 0 & \frac{5+4s}{20s} \end{array} \right|$$

$$= \frac{5+4s}{20s} \cdot I_{i1s} - 0$$

$$V_{i1s} = \frac{\Delta V_{i1s}}{\Delta} = I_{i1s} \cdot \frac{5+4s}{20s} \times \frac{1280s^2}{768s^3 + 1216s^2 + 320s + 352} \dots$$

$$\frac{V_{i1s}}{I_{i1s}} = \frac{5+4s (1280s^2)}{20s (768s^3 + 1216s^2 + 320s + 352)}$$

$$= \frac{320 + 256s}{768s^3 + 1216s^2 + 320s + 352}$$

$$\div 80 \quad \frac{768s^3 + 1216s^2 + 320s + 352}{768s^3 + 1216s^2 + 320s + 352}$$

$$= \frac{3 \cdot 2s + 4}{9.6s^3 + 15 \cdot 2s^2 + 4s + 4.4}$$

$$9.6s^3 + 15 \cdot 2s^2 + 4s + 4.4$$

$$\Delta V_0(s) = \begin{vmatrix} \frac{12s^2 + 4s + 3}{4s} & I_i(s) \\ -\frac{1}{4s} & 0 \end{vmatrix}$$

16

$$= - \left(-\frac{1}{4s} I_i(s) \right) = \frac{I_i(s)}{4s}$$

$$V_0(s) = \frac{\Delta V_0(s)}{\Delta}$$

$$= \frac{I_i(s)}{4s} \times \frac{1280s^2}{768s^3 + 1216s^2 + 320s + 352}$$

$$\frac{V_0(s)}{I_i(s)} = \frac{1280s^2}{4s(768s^3 + 1216s^2 + 320s + 352)}$$

$$= \frac{320s}{768s^3 + 1216s^2 + 320s + 352}$$

$$\div 80$$

$$= \frac{4s}{9.6s^3 + 15.2s^2 + 4s + 4.4}$$

Find the poles and zeros and the scale factor of the given Network. (1)

$$N(s) = \frac{2s+1}{4(s^2+5s+6)}$$

$$4(s^2+5s+6)$$

How many zeros and poles are at infinity?

Draw Pole-zero plot/graph. Check the stability of the given N/w function.

$$N(s) = \frac{2s+1}{4(s^2+5s+6)}$$

$$= \frac{2(s + 1/2)}{4(s^2+5s+6)}$$

$$K = 2/4 = 0.5$$

Zeros $\Rightarrow$ Roots of N_r .

$$2s+1 \Rightarrow 2s = -1 \Rightarrow s = -1/2 = -0.5$$

Poles $\Rightarrow$ Roots of D_r

$$4(s^2+5s+6) = 0.$$

$$s = -2, s = -3$$

$$\text{No. of zeros} = M = 1$$

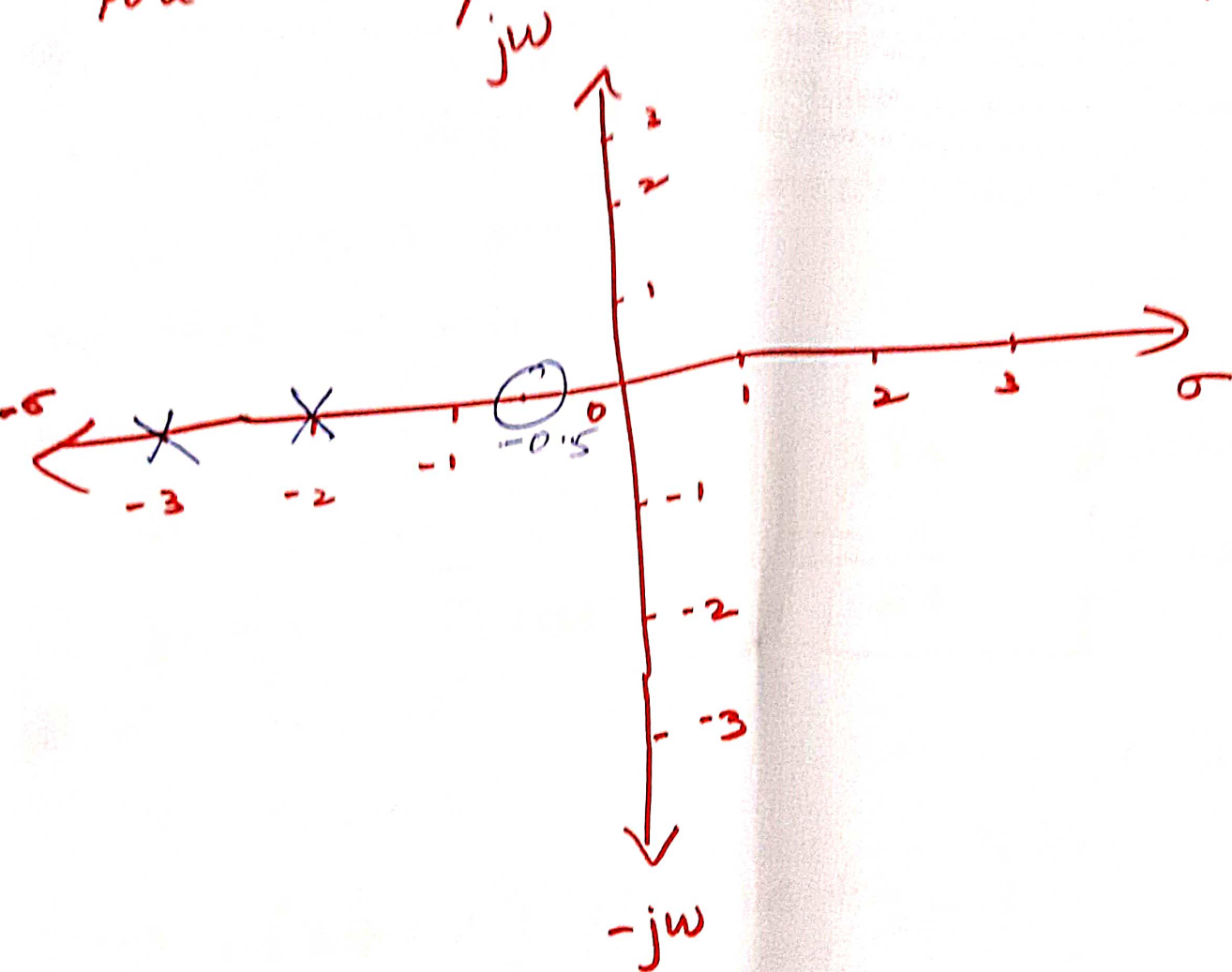
$$\text{No. of poles} = N = 2$$

Zeros at infinity = $N - M = 2 - 1 = 1$

Poles at infinity = $M - N = 1 - 2 = -1$ 2

If $N > M$, zeros at infinity Not applicable

Pole-zero plot is drawn in s-plane



s-plane

x-axis $\Rightarrow \sigma$ = real part of root

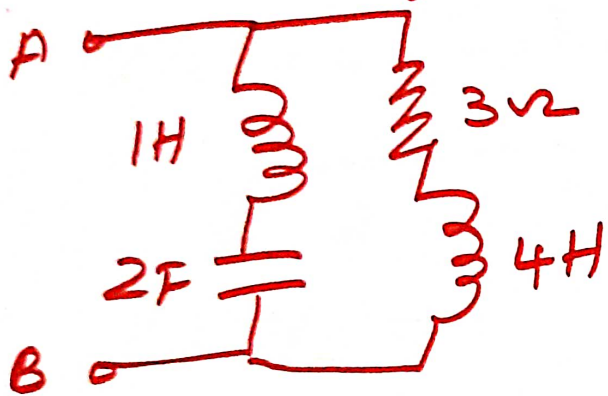
y-axis $\Rightarrow j\omega$ = imaginary part of root

Mark zero as "o" in graph

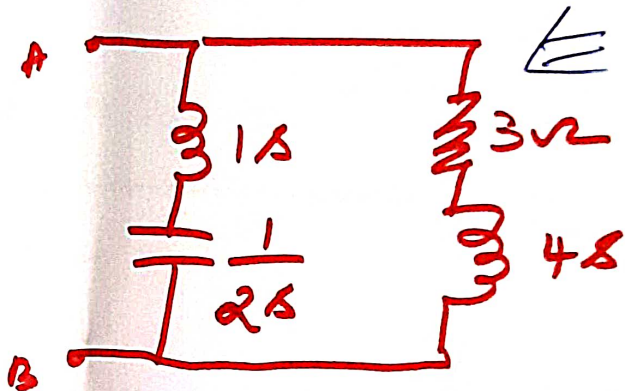
Mark poles as "x" in graph

If all poles and zeros lie on the left hand side (LHS) of s -plane, the network function is stable. 3
 The given function is stable.

Find the driving point impedance across AB for the given circuit. Draw pole-zero graph. Check the stability of the system.



't' domain



s -domain

$$Z(s) = \underbrace{(3 + 4s)}_{\text{series}} \parallel \underbrace{\left(3\Omega + \frac{1}{2s}\right)}_{\text{series}}$$

parallel

$$Z(s) = (3 + 4s) \left(s + \frac{1}{2s}\right)$$

$$\frac{(3 + 4s) + \left(s + \frac{1}{2s}\right)}{(3 + 4s) + \left(s + \frac{1}{2s}\right)}$$

$$\left[\frac{R_1 R_2}{R_1 + R_2} \right]$$

$$= \frac{8s^3 + 6s^2 + 4s + 3}{10s^2 + 6s + 1} = Z(s)$$

(4)

$$Z(s) = \frac{N(s)}{D(s)} \Rightarrow \text{Network function}$$

For this N/w function ie $Z(s)$, find poles, zeros, scale factor.

Plot pole-zero graph.

Check system stability.

$$Z(s) = \frac{2 [4s^3 + 3s^2 + 2s + 1.5]}{2 [5s^2 + 3s + 0.5]}$$

$$K = 2/2 = 1$$

Zeros: Roots of $8s^3 + 6s^2 + 4s + 3$

$$\text{Zeros} = -3/4, 0.707i, -0.707i$$

$$M = 3$$

Poles: Roots of $10s^2 + 6s + 1$

$$\text{Poles} = -0.3 + 0.1i, -0.3 - 0.1i$$

$$N = 2$$

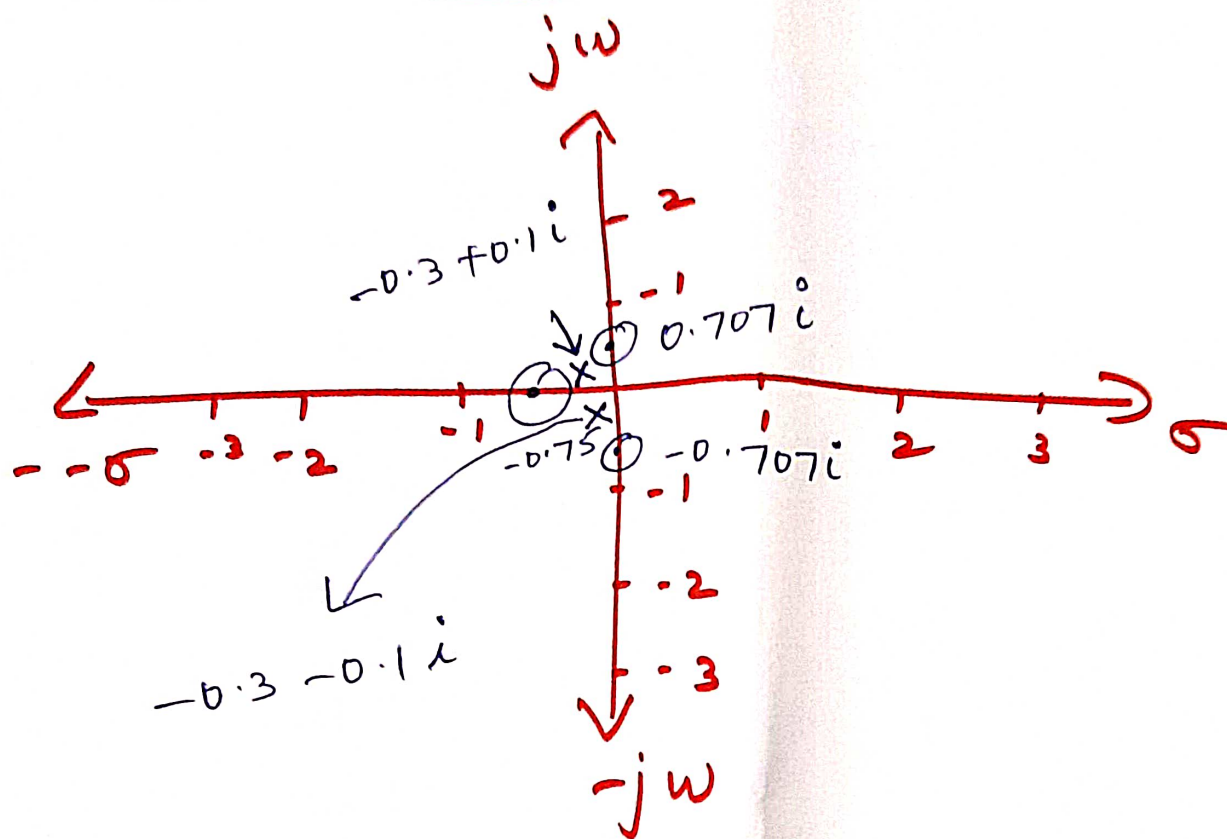
$M > N \Rightarrow$ there are $(M-N)$ poles at infinity.

$$\therefore \text{No. of poles at infinity} = M - N$$

$$= 3 - 2 = 1$$

(4)

pole-zero graph



s-plane

All poles and zeros lie in LHS of s-plane.

$\therefore$ The given Network function/system is stable.

Module - 3

(1)

Transform a circuit in 't' domain into a circuit in 's' domain and vice versa

Circuit consists of power supply (energy source) and elements (R, L, C)

Resistor (R) $\rightsquigarrow$

$$V_t = I_t R_t$$

$t \rightarrow t$ domain

$V_R(t)$ = Voltage across Resistor in 't' domain

$$V_R(t) = I_R(t) \cdot R_t$$

$$I_R(t) = \frac{V_R(t)}{R_t}$$

$$V_R(s) = I_R(s) \cdot R_s$$

$$R_t = R_s = R$$

$$V_R(s) = I_R(s) R$$

$$I_R(s) = \frac{V_R(s)}{R}$$
$$= G V_R(s)$$

$V_R(s)$ = Voltage across Resistor in 's' domain

$$\frac{1}{R} = G$$

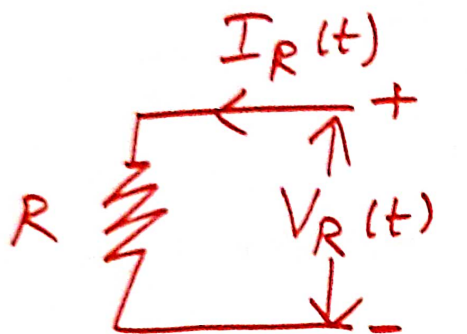
Conductance

$$\text{Impedance} = \frac{V}{I} = Z$$

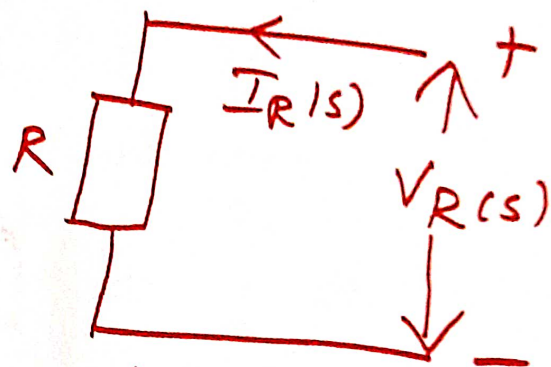
$$\text{Admittance} = \frac{I}{V} = Y$$

$$\text{Transform Impedance} = \frac{V_R(s)}{I_R(s)} = Z_R(s)$$

$$\text{Transform Admittance} = \frac{I_R(s)}{V_R(s)} = Y_R(s)$$



't' domain



's' domain

Inductor L - (H) - Henry

$$V_L(t) = L \frac{di(t)}{dt}$$

$$I_L(t) = \frac{1}{L} \int_{-\infty}^t V_L(t) dt$$

Transform Equation

(3)

$$V_L(s) = L [s I_L(s) - i_L(0)]$$

$$= L s I_L(s) - L i_L(0)$$

$$V_L(s) + L i_L(0) = L s I_L(s)$$

$$\downarrow$$
$$V_L(s) = L s I_L(s)$$

Transform Impedance

$$Z_L(s) = \frac{V_L(s)}{I_L(s)} = \frac{L s I_L(s)}{I_L(s)} = L s$$

$L \rightarrow$ value of the inductor.

$$I_L(s) = \frac{1}{L} \left[\frac{V_L(s)}{s} + \frac{\int_{-\infty}^{0^+} V_L(t) dt}{s} \right]$$

$$\int_{-\infty}^{0^+} V_L(t) dt = L i_L(0^+)$$

$$I_L(s) = \frac{1}{L} \left[\frac{V_L(s)}{s} + \frac{L i_L(0^+)}{s} \right]$$

$$= \frac{V_L(s)}{L s} + \frac{i_L(0^+)}{s} = \underline{I_L(s)}$$

$$= \frac{V_L(s)}{Ls} = I_L(s) - \frac{i_L(0^+)}{s}$$

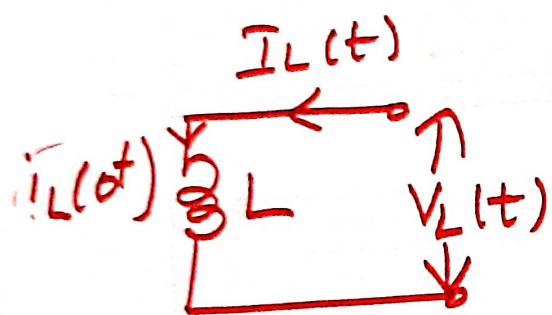
(4)

$$I_L(s) - \frac{i_L(0^+)}{s} = I_1(s)$$

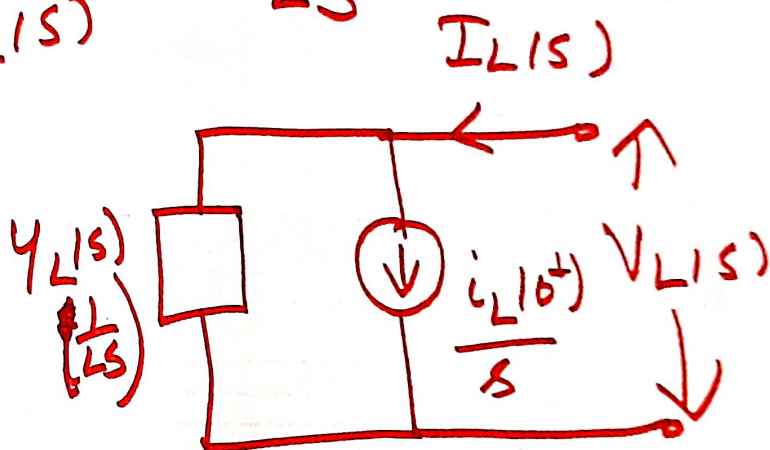
$$\frac{V_L(s)}{Ls} = I_1(s)$$

Transform Admittance

$$Y_L(s) = \frac{I_1(s)}{V_L(s)} = \frac{1}{Ls}$$



't' domain



's' domain

Capacitor 'C' (Farad)

$$V_C(t) = \frac{1}{C} \int_{-\infty}^{\infty} I_C(t) dt$$

$$I_C(t) = C \frac{dV_C(t)}{dt}$$

Transform Equation

$$V_C(s) = \frac{1}{C} \left[\frac{I_C(s)}{s} + \frac{q(0^+)}{s} \right] \quad (5)$$

$$\frac{q(0^+)}{s} = V_C(0^+)$$

$$= \frac{1}{C} \left[\frac{I_C(s)}{s} + V_C(0^+) \right]$$
$$= \frac{I_C(s)}{Cs} + \frac{V_C(0^+)}{s}$$

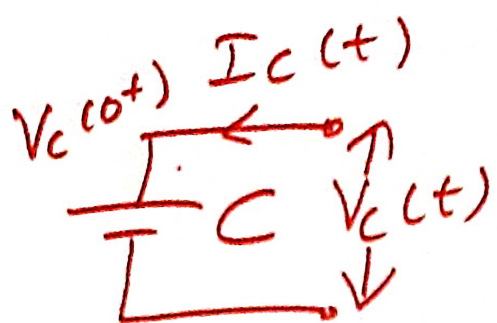
$$\frac{I_C(s)}{Cs} = V_C(s) - \frac{V_C(0^+)}{s}$$

$$V_C(s) - \frac{V_C(0^+)}{s} = V_1(s)$$

$$\frac{I_C(s)}{Cs} = V_1(s)$$

Transform impedance

$$Z_C(s) = \frac{V_1(s)}{I_C(s)} = \frac{1}{Cs}$$



$\Rightarrow \frac{1}{Cs}$



$$I_C(s) = C [s V_C(s) - V_C(0+)]$$

$$= Cs V_C(s) - C V_C(0+) = I_C(s)$$

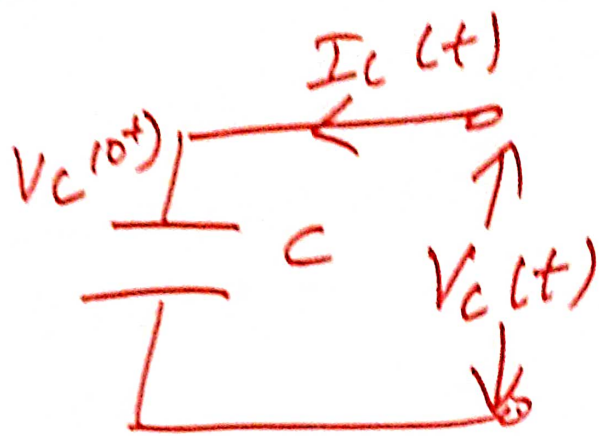
$$Cs V_C(s) = I_C(s) + C V_C(0+)$$

$$I_C(s) + C V_C(0+) = I_1(s)$$

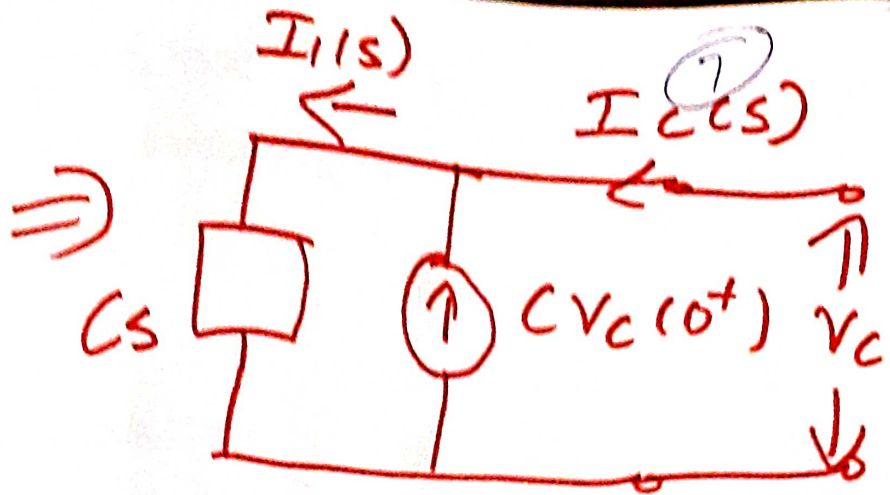
$$Cs V_C(s) = I_1(s)$$

Transform Admittance

$$Y_C(s) = \frac{I_1(s)}{V_C(s)} = Cs$$

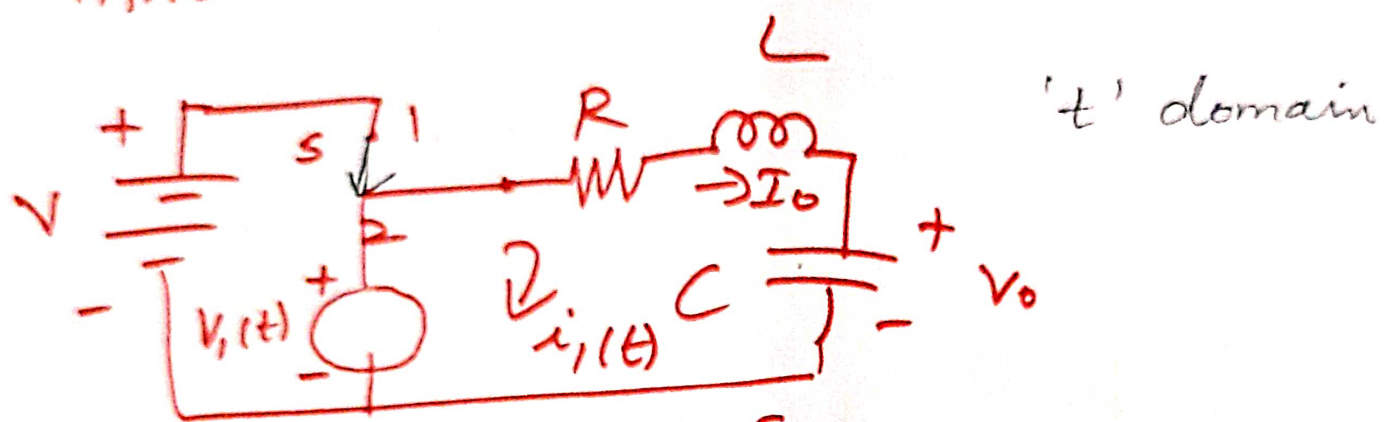


't' domain



's' domain

In the ckt given, Switch S is moved from position '1' to position '2' at time $t=0$. At time $t=0^-$, the current thro' Inductor L is I_0 and the Voltage across capacitor is V_0 . Find the transform current $I_1(s)$



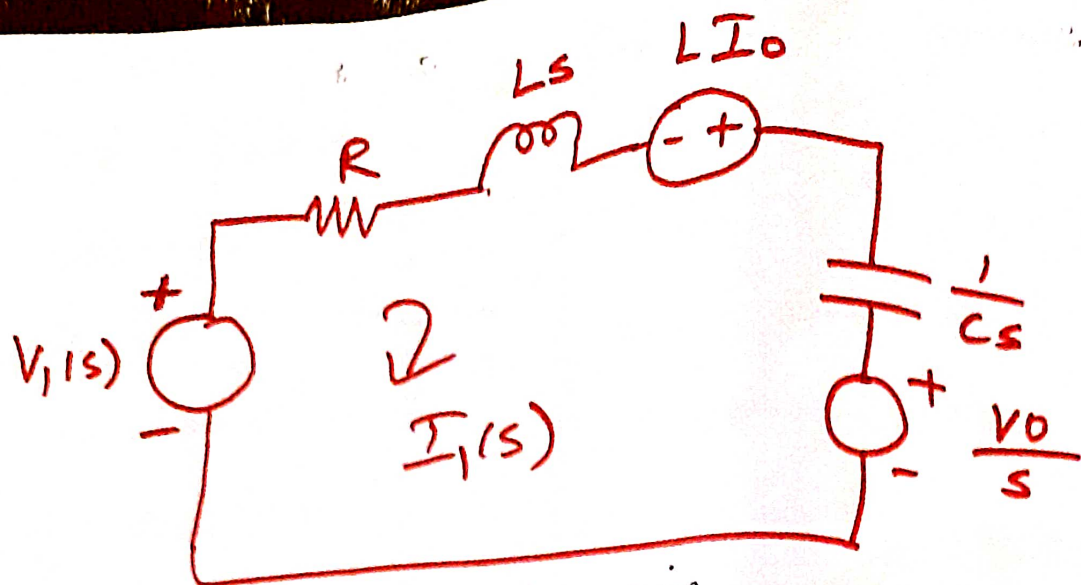
When S is moved from position 1 and to position 2, second loop of the circuit is closed (closed loop)

Given circuit is a series circuit.

$\therefore$ we have to find Transfer Impedance.

Given circuit is in 't' domain

To find Transfer Impedance, Convert circuit in 't' domain into circuit in 's' domain



Circuit in 's' domain

Calculate Transfer Impedance using this circuit in 's' domain

Transfer Impedance $Z(s) = \frac{V(s)}{I_1(s)}$

$V(s)$ = algebraic potential drop in the circuit

$$V(s) = V_1(s) + L I_0 - \frac{V_0}{s}$$

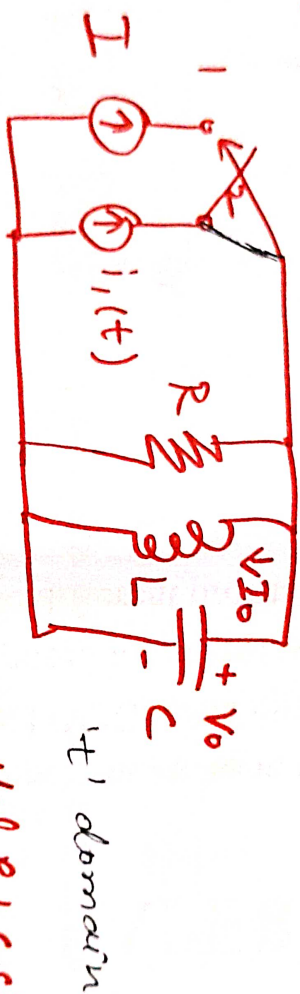
Consider the leaving terminal of $I_1(s)$

$$Z(s) = R + Ls + \frac{1}{Cs}$$

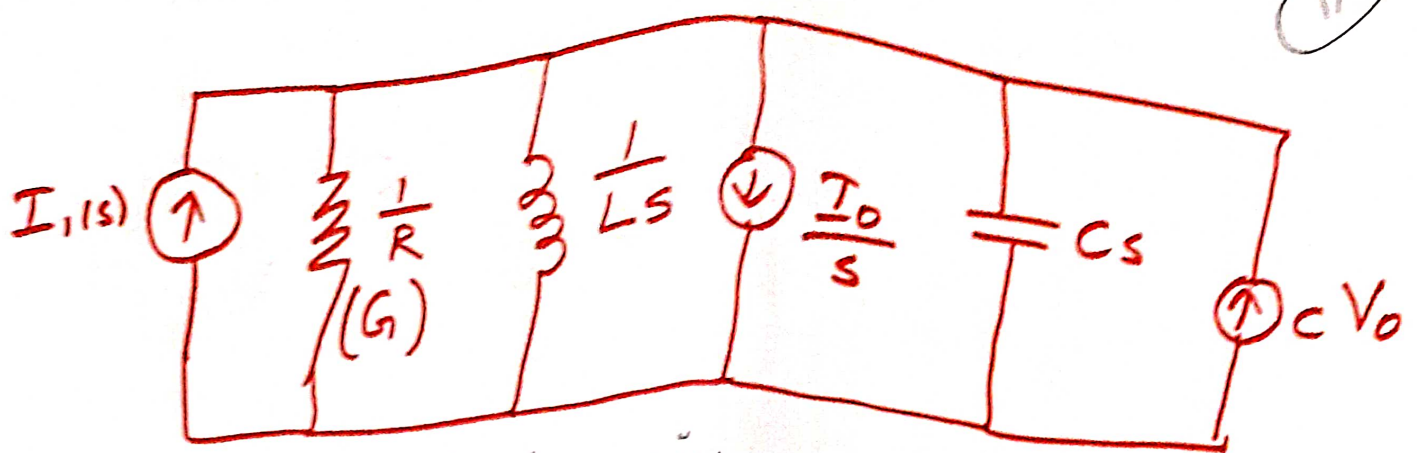
$Z(s)$ = Sum of all impedances connected in series
(R, L, C)

$$I_1(s) = \frac{V(s)}{Z(s)} = \frac{V_1(s) + L I_0 - V_0/s}{R + Ls + 1/Cs}$$

The given network is a parallel combination of L , R , C connected across a current source I . At time $t = 0^-$, the current through inductor L is I_0 and the voltage across capacitor C is V_0 . At time $t = 0^+$, the current source I , $i(t)$ is connected to the parallel RLC circuit. Find the transform voltage $V(s)$.



Given circuit is a parallel RLC circuit. $\therefore$ we have to find Transform Admittance. To find Transform Admittance, we need circuit in ' s ' domain.



Circuit in 's' domain.

Transform Admittance $Y(s) = \frac{I(s)}{V(s)}$

$$= \frac{I(s)}{V(s)}$$

$$Y(s) = \frac{1}{R} + \frac{1}{Ls} + Cs = G + \frac{1}{Ls} + Cs$$

$Y(s)$ is the sum of all impedances connected in parallel

↓ because arrow is downward.

$$I(s) = I_1(s) - \frac{I_0}{s} + CV_0$$

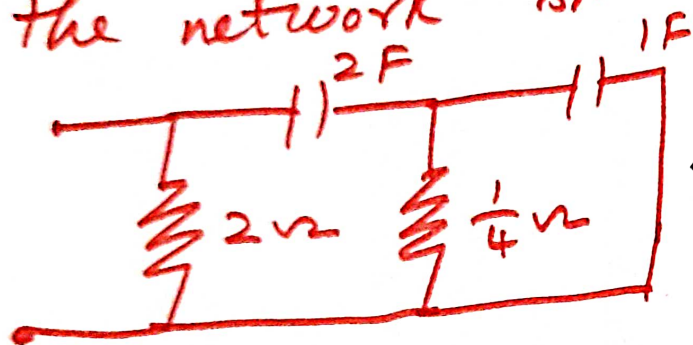
$I(s)$ = sum of all current sources in circuit.

$$V(s) = \frac{I(s)}{Y(s)} = \frac{I_1(s) - I_0/s + CV_0}{G + \frac{1}{Ls} + Cs}$$

Series	parallel	element
R	$\frac{1}{R}$	R
LS	$\frac{1}{LS}$	L
$\frac{1}{CS}$	CS	C

12

Obtain the transform impedance of the network shown in figure.



start calculating the impedance

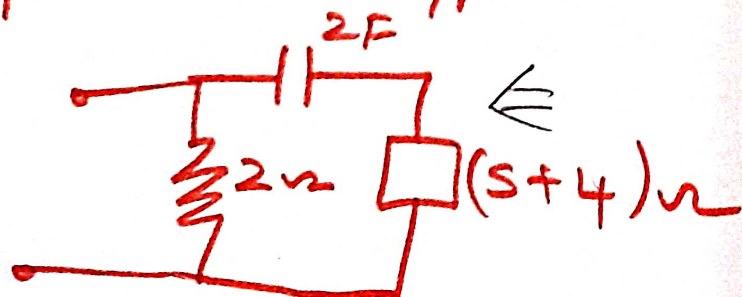
Starting point of N/w

1F and $\frac{1}{4} \Omega$ are parallel

$$CS = s \quad \frac{1}{R} = \frac{1}{\frac{1}{4}} = 4$$

$$(s+4)$$

parallel $\rightarrow$ o/p $\rightarrow$ Transform admittance $\hookrightarrow (s+4) \Omega$



$2F$ and $(s+4)$ are in series

13

$$2F \Rightarrow \frac{1}{Cs} = \frac{1}{2s}$$

$(s+4)$ is Transform admittance.

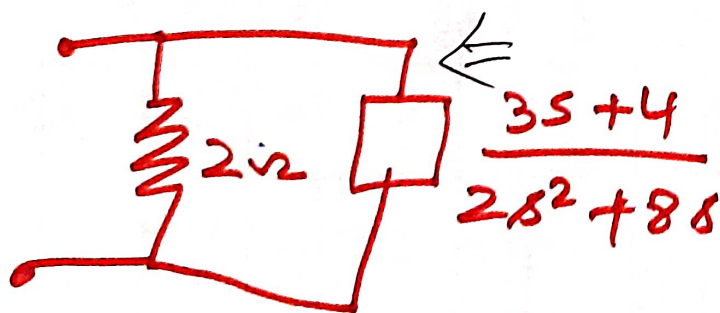
In series we need Transform impedance.
So convert Transform admittance into Transform impedance.

$$Y(s) = \frac{1}{s+4} \quad \left[\begin{array}{l} Y(s) = \frac{1}{2(s)} \\ 2(s) = \frac{1}{Y(s)} \end{array} \right]$$

$$2(s) = \frac{1}{s+4} \Rightarrow \text{Transform impedance}$$

$\frac{1}{2s}$ series with $\frac{1}{s+4}$

$$\frac{1}{2s} + \frac{1}{s+4} = \frac{(s+4) + 2s}{2s(s+4)} = \frac{3s+4}{2s^2+8s}$$



Transform impedance $2(s)$

14

$$2r = \frac{1}{2}$$

$2r$ and $\frac{3s+4}{2s^2+8s}$ are in parallel

$$2r = \frac{1}{R} = \frac{1}{2}$$

$$2(s) \Rightarrow 4(s)$$

$$4(s) = \frac{2s^2+8s}{3s+4}$$

$$\frac{1}{2} \parallel \frac{2s^2+8s}{3s+4}$$

$$\frac{1}{2} \times \frac{2s^2+8s}{3s+4}$$

$$= \frac{2s^2+8s}{6s+8}$$

$$\frac{1}{2} + \frac{2s^2+8s}{3s+4}$$

$$\frac{(3s+4) + 4s^2 + 16s}{2(3s+4)}$$

$$= \frac{(2s^2+8s) \times (6s+8)}{2(3s+4)}$$

$$4s^2 + 19s + 4$$

$$= \frac{12s^3 + 48s^2 + 16s^2 + 64s}{4s^2 + 19s + 4}$$

$$4s^2 + 19s + 4$$

$$= \frac{12s^3 + 64s^2 + 64s}{4s^2 + 19s + 4} = 4(s)$$

(15)

Transform Impedance $Z(s)$

$$= \frac{4s^2 + 19s + 4}{12s^3 + 64s^2 + 64s}$$

Module 4

①

Resonance

Resonant frequency

$$f_0, f_r = \frac{1}{2\pi\sqrt{LC}}$$

Hertz (Hz)

At resonance

$$X_L = X_C, Z = R$$

$$X_L \propto f$$

$$X_C \propto \frac{1}{f}$$

$$X_L = \text{Inductive reactance} = 2\pi f L$$

$L \rightarrow$ value of inductor in Henry

$f \rightarrow$ frequency of supply = 50 Hz.

$$X_C = \text{Inductive capacitance} = \frac{1}{2\pi f C}$$

$C \rightarrow$ value of capacitor in Farad

$$\text{Impedance } Z = R + jX$$

$X =$ net reactance of the circuit

$$X = X_L - X_C$$

$$\text{Voltage across Inductor } V_L = I X_L$$

$$\text{Voltage across Capacitor } V_C = I X_C$$

$$\text{Voltage across Resistor } V_R = I R$$

(2)

Current in a resonant circuit

$$I = \frac{V}{R}$$

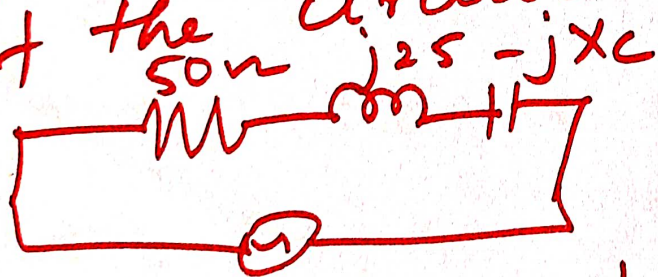
Quality factor of a coil

$$Q = \frac{\text{Resonant frequency}}{\text{Band width}}$$

$$\text{Band width } BW = (f_2 - f_1) \text{ Hz}$$

$$Q = \frac{L}{R} = \frac{X_L}{R}$$

For the given ^{ckt} determine the value of capacitive reactance and impedance of the circuit.



$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times C}$$

At resonance $X_L = X_C$

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times j25$$

$$X_L = 7850 = 2 \times 3.14 \times 50 \times 25$$

3

$$X_L = X_C = 7850$$

$$X_C = 7850 = \frac{1}{2\pi f C}$$

$$\frac{1}{2\pi f C} = 7850$$

$$2\pi f C = \frac{1}{7850}$$

$$C = \frac{1}{7850 \times 2 \times 3.14 \times 50}$$

$$= \frac{1}{2464900} = 0.4 \times 10^{-6} \text{ F}$$

$$= 0.4 \mu \text{ F}$$

$$X_C = X_L = 7850 \text{ F}$$

Z = Impedance of the circuit.

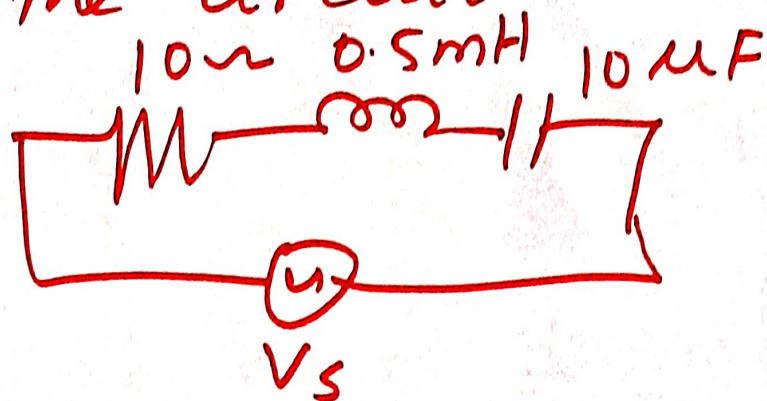
$$Z = R + jX$$

$$= R + j(X_L - X_C)$$

$$Z = R = 50 \Omega$$

Determine the resonant freq for the circuit

(4)

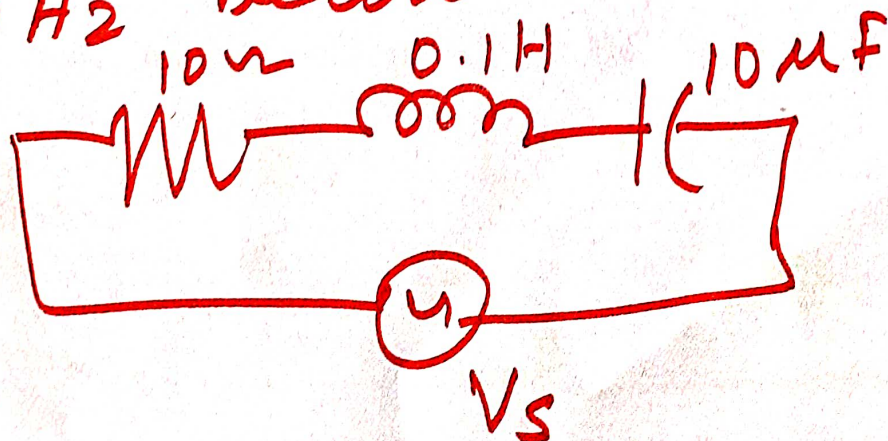


$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \sqrt{0.5 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$$= 2.25 \text{ KHz.}$$

Determine impedance at resonant freq, 10Hz above resonant freq, 10Hz below resonant freq.



Resonant freq

(5)

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \sqrt{0.1 \times 10 \times 10^{-6}}}$$
$$= 159.2 \text{ Hz.}$$

10 Hz above resonant freq

$$159.2 + 10 = 169.2 \text{ Hz.}$$

10 Hz below resonant freq

$$159.2 - 10 = 149.2 \text{ Hz.}$$

At resonance frequency (159.2 Hz)

$$Z = R = 10 \Omega$$

Impedance at 169.2 Hz.

$$Z = R + jX \Omega$$

$$X = X_L - X_C$$

calculate X_L and X_C at 169.2 Hz.

$$X_L = 2\pi f L = 2 \times 3.14 \times 169.2 \times 0.1 = 106.26 \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 169.2 \times 10 \times 10^{-6}}$$
$$= ~~106.26 \Omega~~ 94.11$$

6

$$X = X_L - X_C = 106.26 - 94.11 \\ = 12.15$$

$$Z = R + jX \Omega = 10 + j12.15 \Omega.$$

Impedance at 169.2 Hz .

Impedance at 149.2 Hz .

$$Z = R + jX \Omega$$

$$X = X_L - X_C$$

$$X_L = 2\pi fL = 2 \times 3.14 \times 149.2 \times 0.1 \\ = 93.698 \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 149.2 \times 10 \times 10^{-6}} \\ = 106.73 \Omega$$

$$X = X_L - X_C = -13.032$$

$$Z = 10 - 13.032j \Omega$$

A series circuit with $R = 10\Omega$, $L = 0.1H$, $C = 50\mu F$ has an $\textcircled{7}$ applied voltage $V = 50\angle 0^\circ$ with a variable frequency. Find the resonant freq, the value of freq at which maximum voltage occurs across the inductor and the value of frequency at which max voltage occurs across the capacitor.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \sqrt{0.1 \times 50 \times 10^{-6}}}$$

$$= 71.18 \text{ Hz}$$

freq at which max voltage appear across Inductor

$$f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 - \frac{R^2 C}{2L}}}$$

$$= 72.08 \text{ Hz}$$

freq at which max voltage appear across capacitor.

(8)

$$f_c = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$= 71.08 \text{ Hz.}$$

Max. voltage appear across Inductor above resonant freq.

Max. voltage appear across capacitor below resonant freq.

Determine Quality factor (Q) of a coil for the series circuit consisting of $R = 10\Omega$, $L = 0.1 \text{ H}$, $C = 10\mu\text{F}$.

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

$$\text{Bandwidth} = f_2 - f_1 \quad (\text{no frequency details provided})$$

$$BW = \frac{R}{2\pi L} = \frac{10}{2 \times 3.14 \times 0.1} = 15.91 \text{ Hz}$$

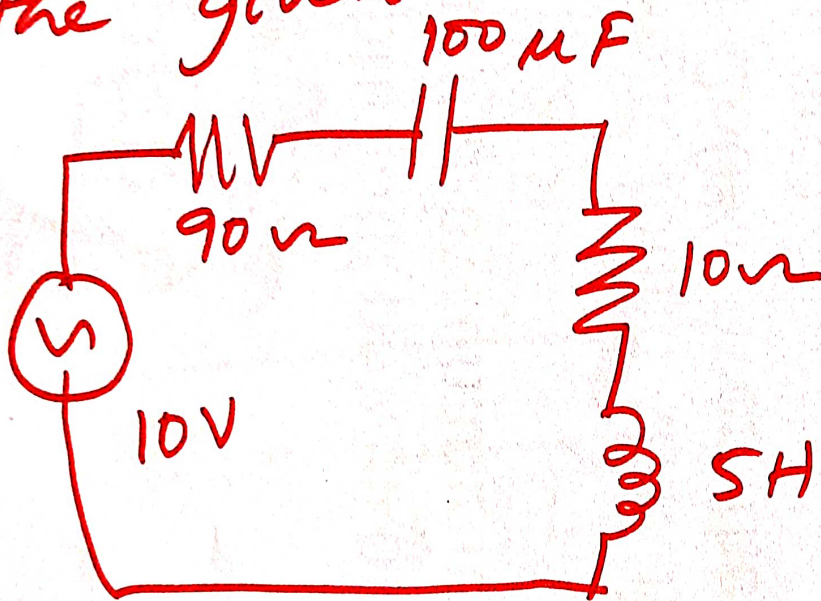
Resonant freq $f_r = \frac{1}{2\pi\sqrt{LC}}$

(9)

$$\frac{1}{2 \times 3.14 \times 0.1 \times 10 \times 10^{-6}} = 159.2 \text{ Hz}$$

$$Q = \frac{159.2}{15.91} = 10 \text{ (no unit)}$$

Determine the value of Q at resonance and bandwidth of the given circuit.



AC
Series
circuit.

$$Q = \frac{f_r}{BW}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \sqrt{5 \times 100 \times 10^{-6}}} = 7.12 \text{ Hz}$$

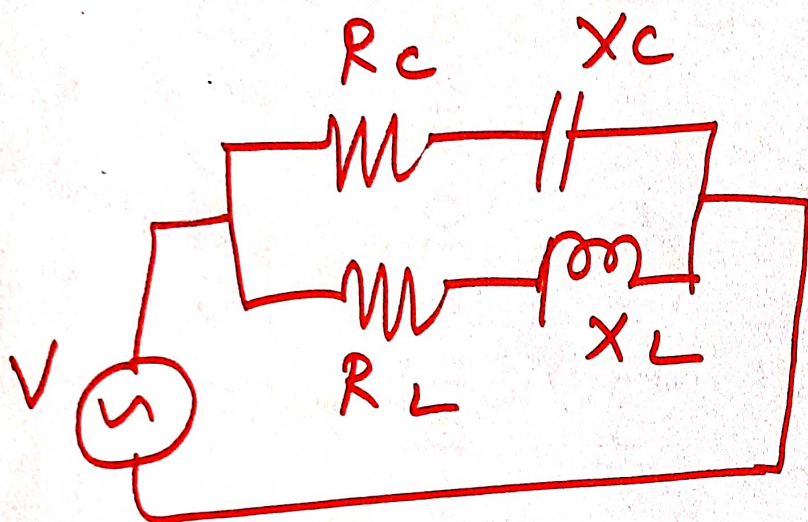
$$BW = \frac{R}{2\pi L} = \frac{90 + 10}{2 \times 3.14 \times 5}$$

10

$$= 3.185 \text{ Hz}$$

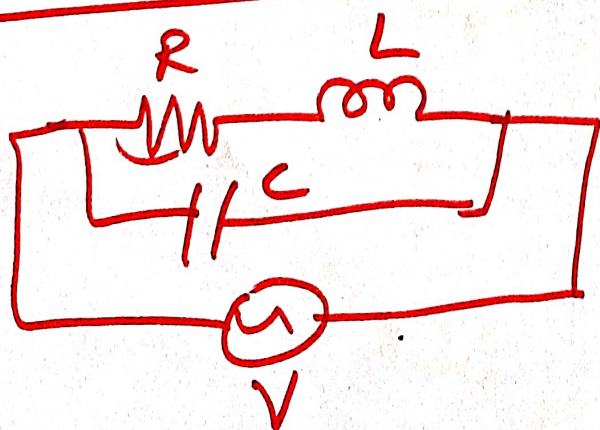
$$Q = \frac{7.12}{3.185} = 2.235 \text{ (no unit)}$$

Parallel Resonance



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

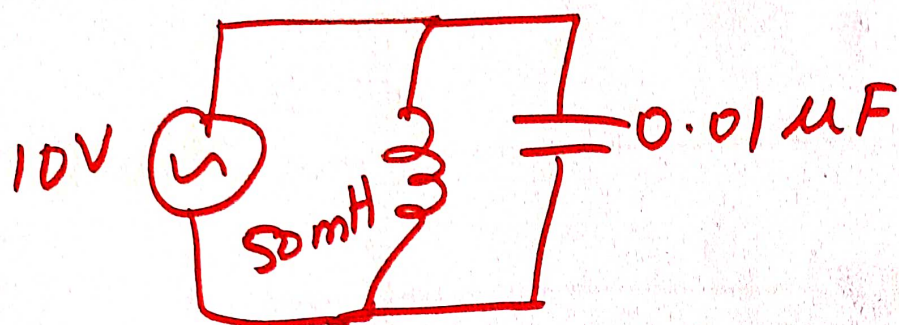
Tank circuit



$$f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 - \frac{R^2}{L^2}}}$$

Find resonant freq in ideal parallel LC circuit.

(11)

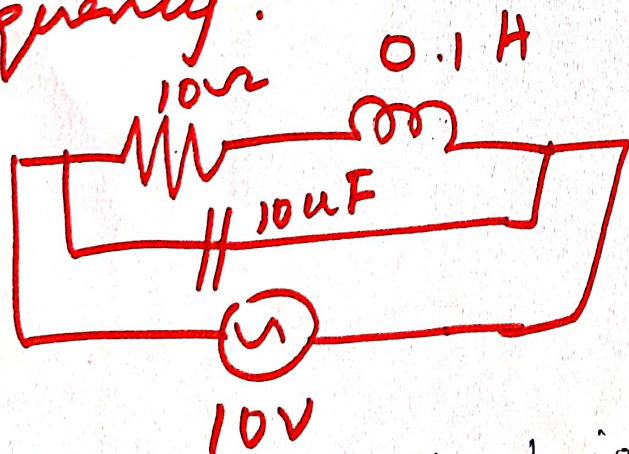


$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50 \times 10^{-3} \times 0.01 \times 10^{-6}}}$$

$$= 7117.6 \text{ Hz.}$$

$$= 7.1 \text{ KHz.}$$

For tank circuit, find resonant frequency.



R, L, C

$$f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$= \frac{1}{2\pi\sqrt{0.1 \times 10^{-6}}} \sqrt{\frac{1}{0.1 \times 10^{-6}} - \frac{10^2}{0.1^2}}$$

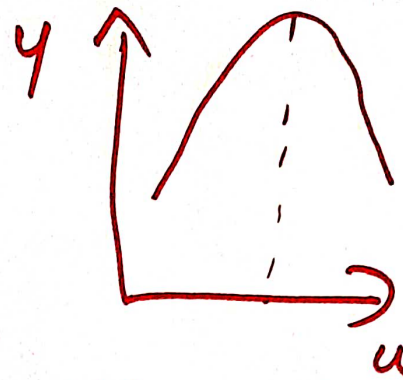
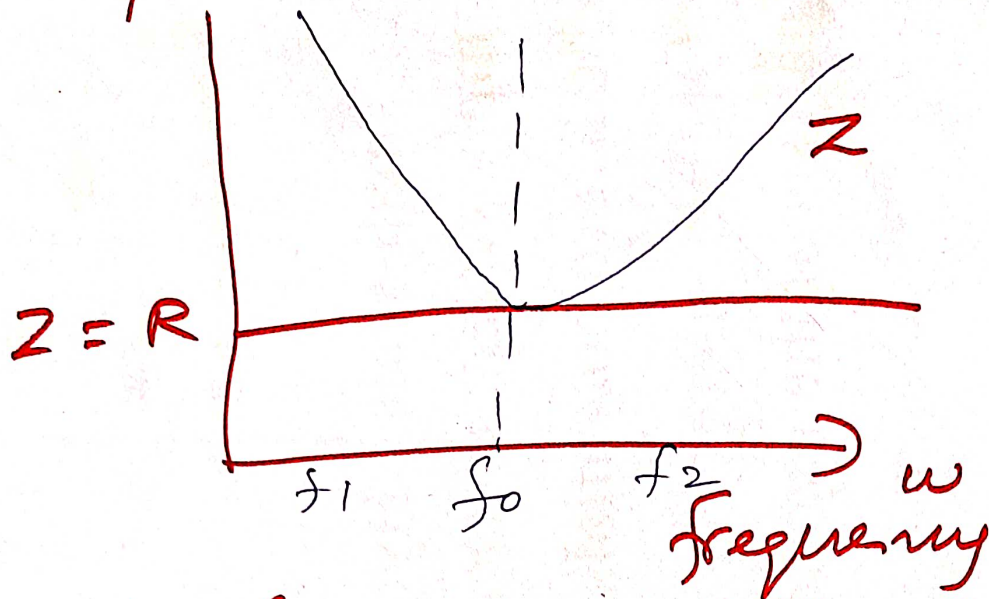
$$R \text{ connected with } L \text{ is } R_L = \frac{1}{2 \times 3.14} \sqrt{10^6 - 10^2}$$

$$= 158.35 \text{ Hz.}$$

Series Resonance

Impedance Vs Frequency

(12)



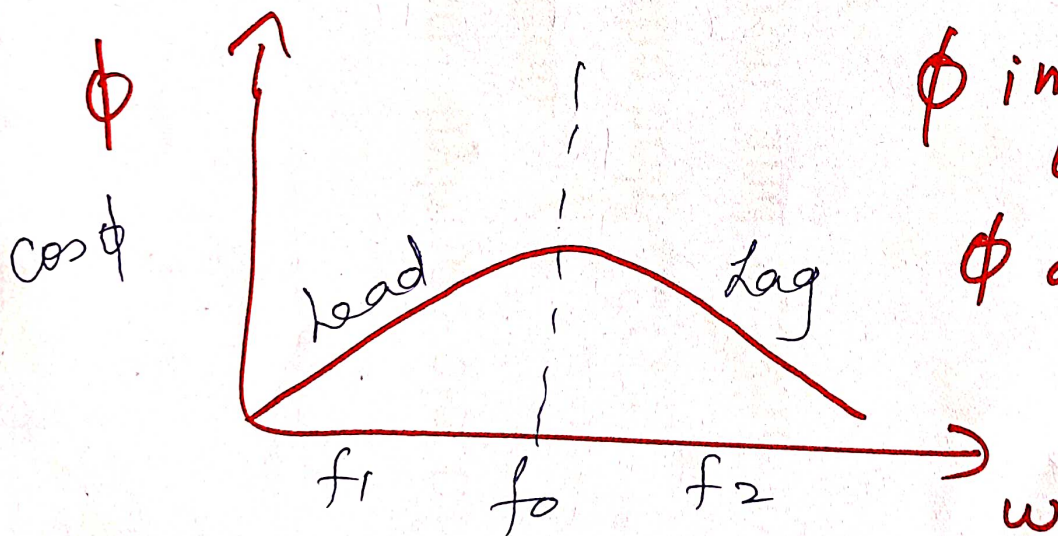
Y increases with freq
 Y decreases with freq.

At Resonance $Z = R$

f_0 = resonant frequency

At lower freq, Z decreases
At higher freq, Z increases.

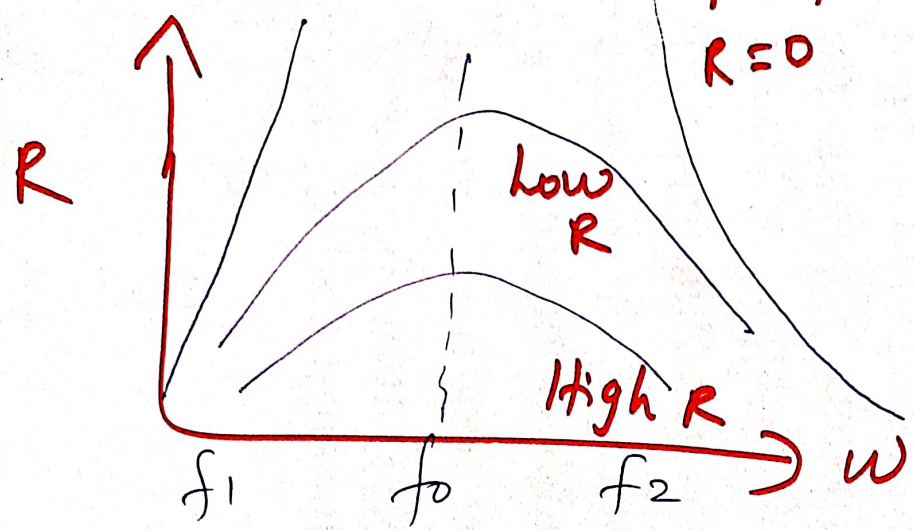
Phase angle Vs Frequency



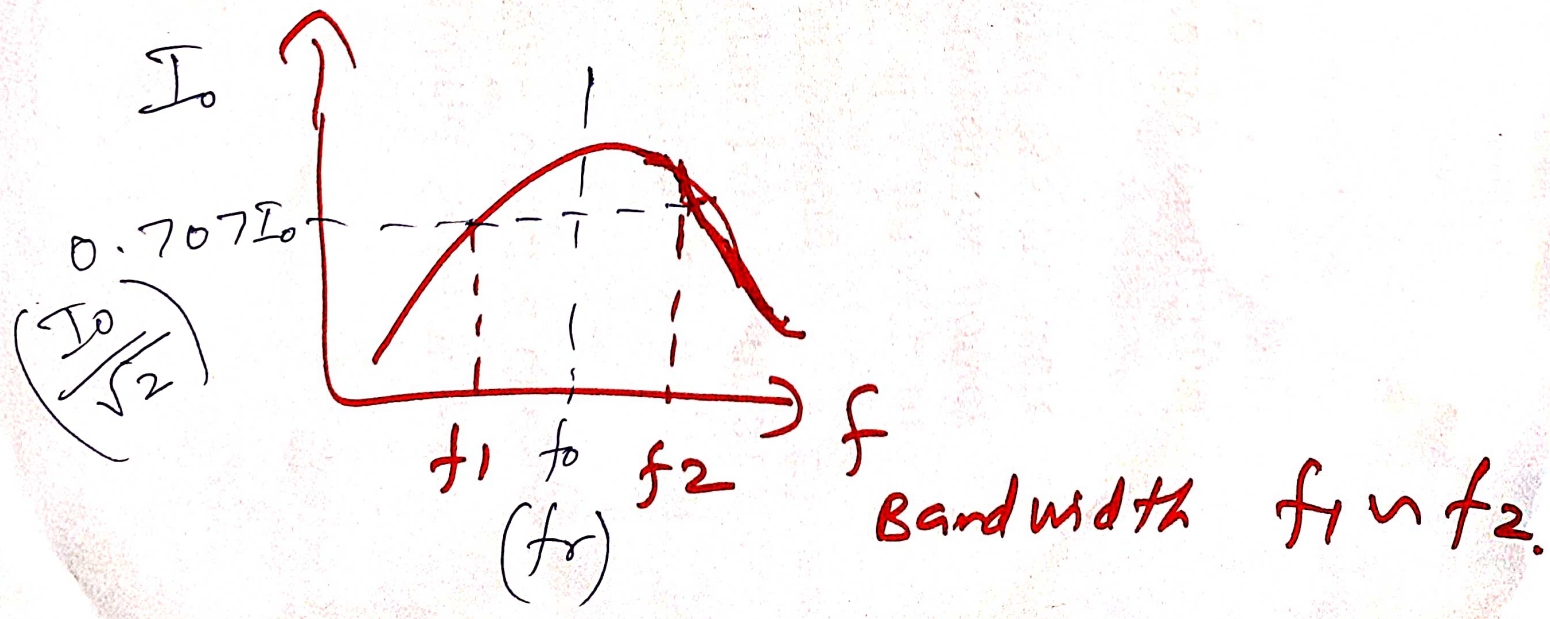
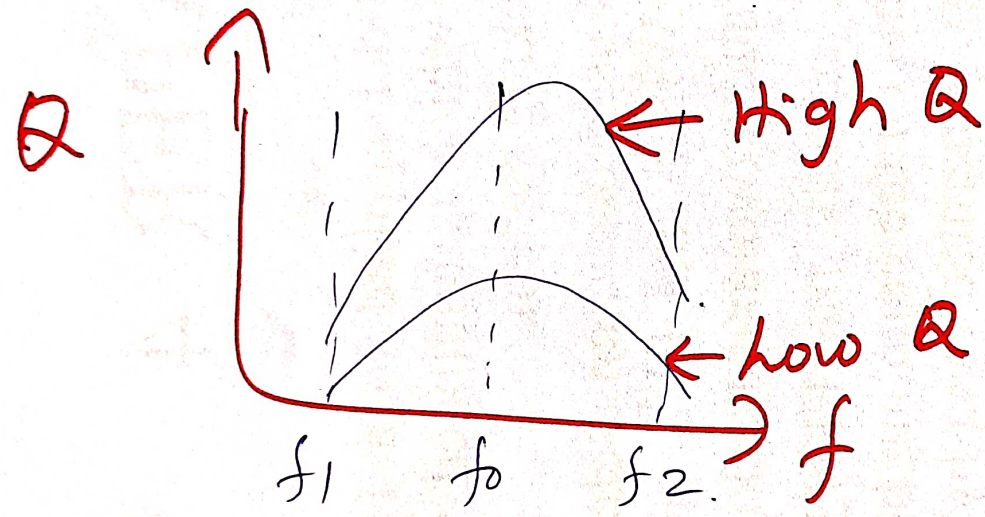
ϕ increases at lower freq
 ϕ decreases at higher freq.

$\cos \phi$ = power factor.

Resistance Vs frequency
↑ infinity
 $R=0$

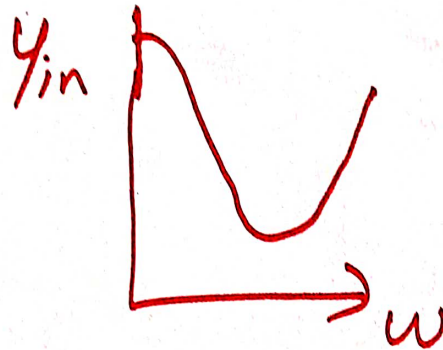
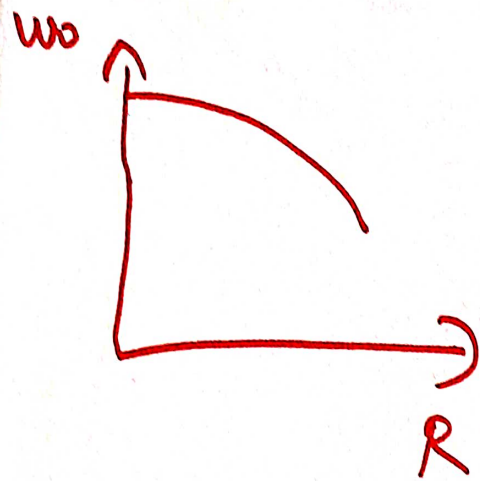


Q factor Vs freq.



Parallel Resonance

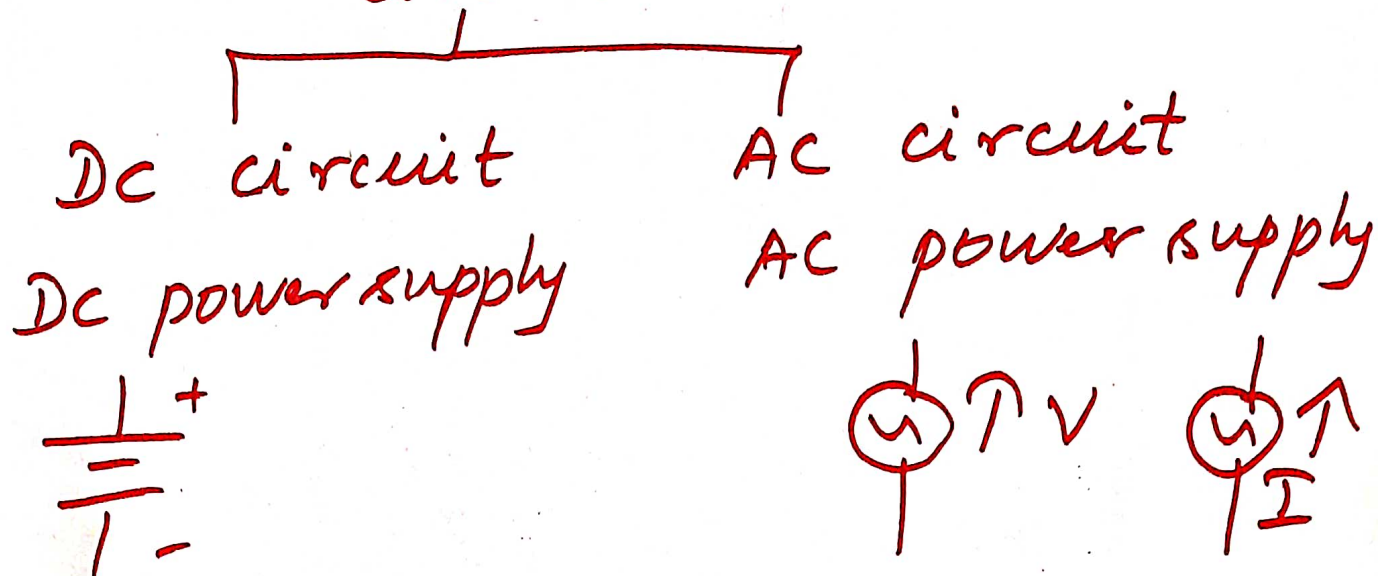
14



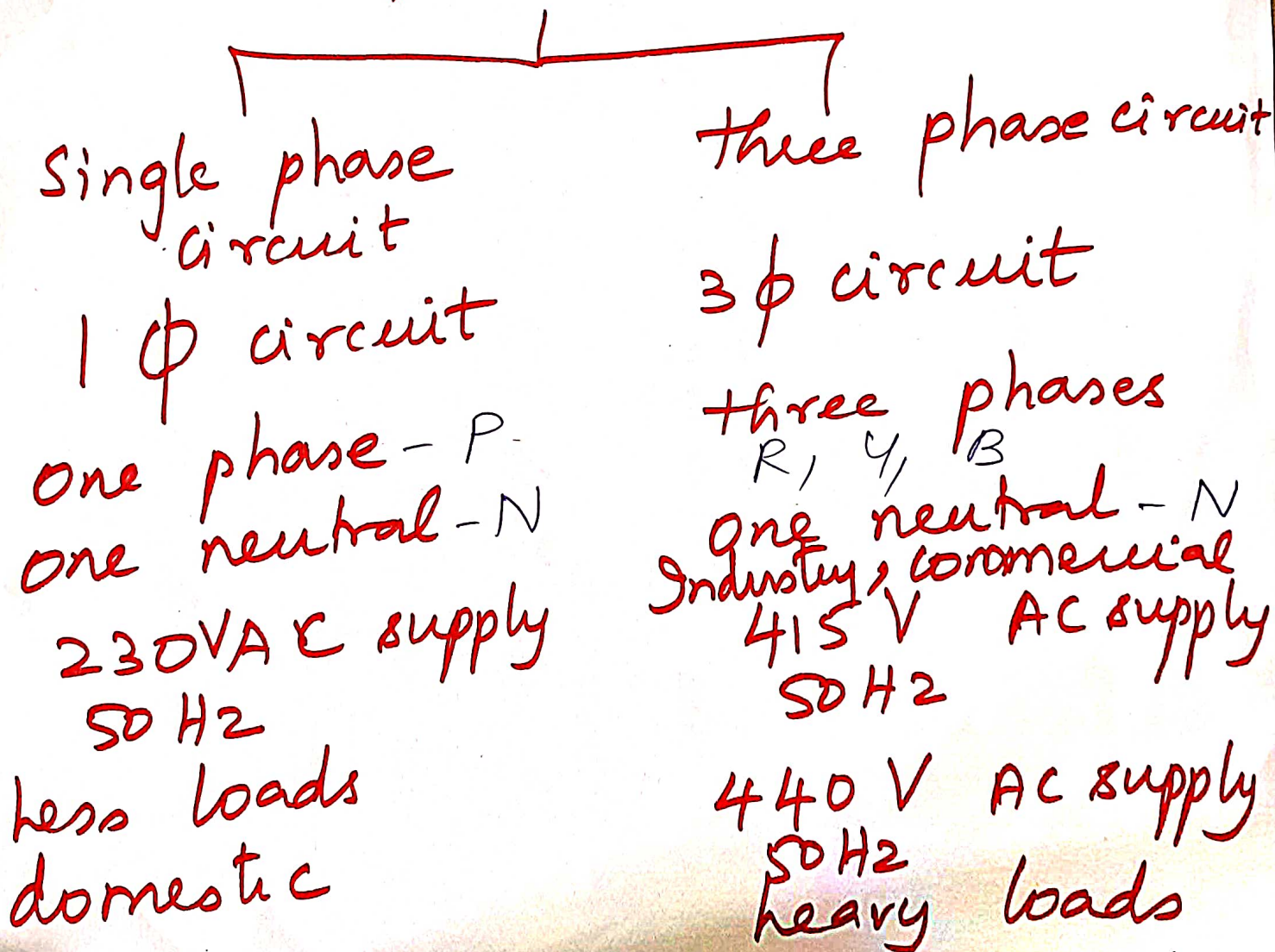
Three phase Networks

(1)

Circuits

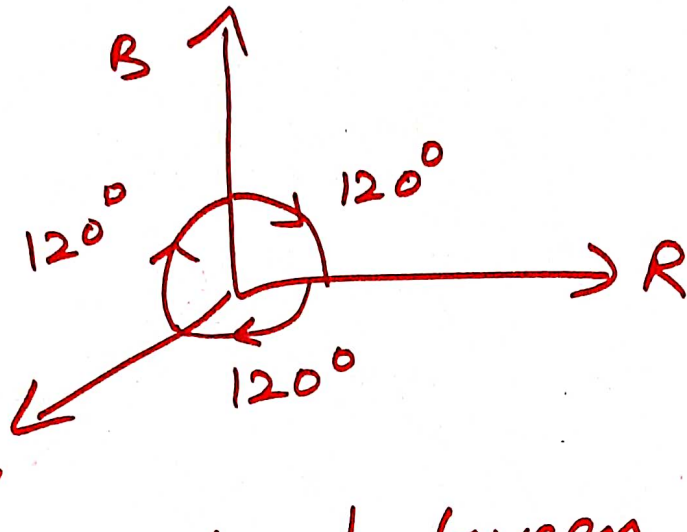


AC circuit



3 ϕ system

3 phases - combination of three
phases R, Y, B
They differ in phase by 120°



phase angle between R & Y is 120°
" " Y & B is 120°
" " B & R is 120°
" " R & B = $120^\circ + 120^\circ$
" " " = 240°

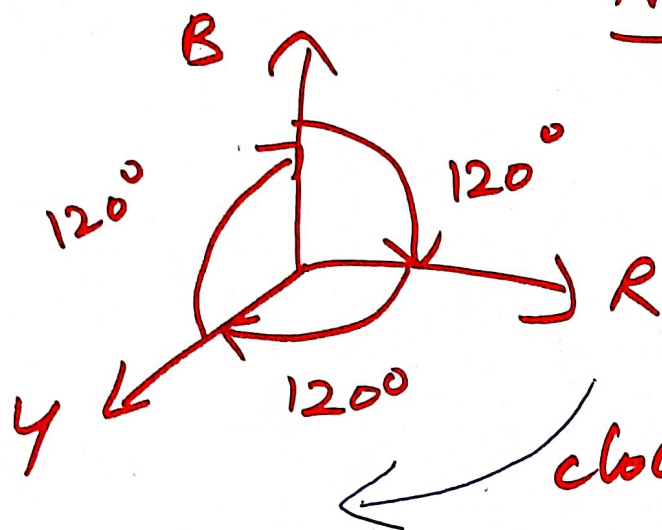
$$B \text{ \& } Y = 120^\circ + 120^\circ = 240^\circ$$

Phase sequence

1. R Y B phase sequence - clockwise direction
2. R B Y phase sequence.
anti clockwise direction.

R Y B phase sequence (or) (3)

Negative phase sequence

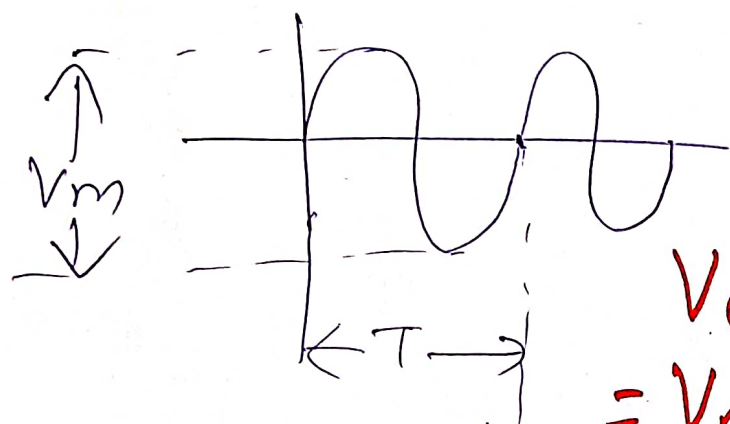


clockwise direction.

$$V_{RR} = E_{RR} = V_m \sin \omega t$$

$$V_{YY} = E_{YY}$$

$$= V_m \sin(\omega t - 120^\circ)$$



T $\rightarrow$ Time period.

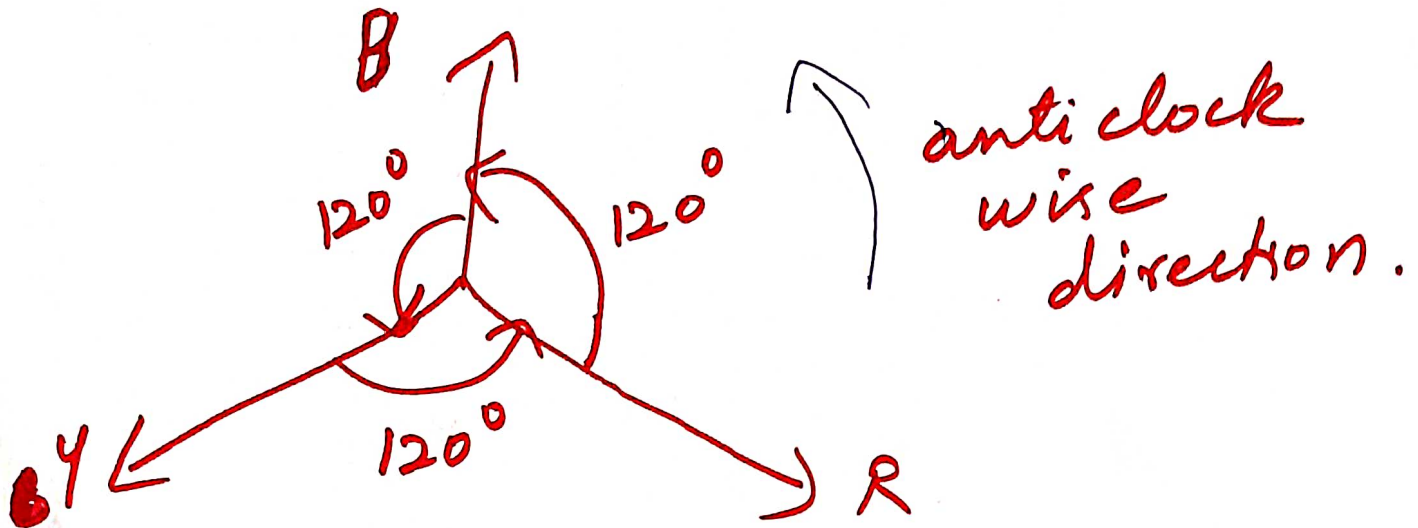
$$V_{BB} = E_{BB} \\ = V_m \sin(\omega t - 240^\circ) \\ \text{(or)}$$

$$= V_m \sin(\omega t + 120^\circ)$$

Most commonly used phase sequence is RYB phase sequence.

RBY phase sequence (or) ④

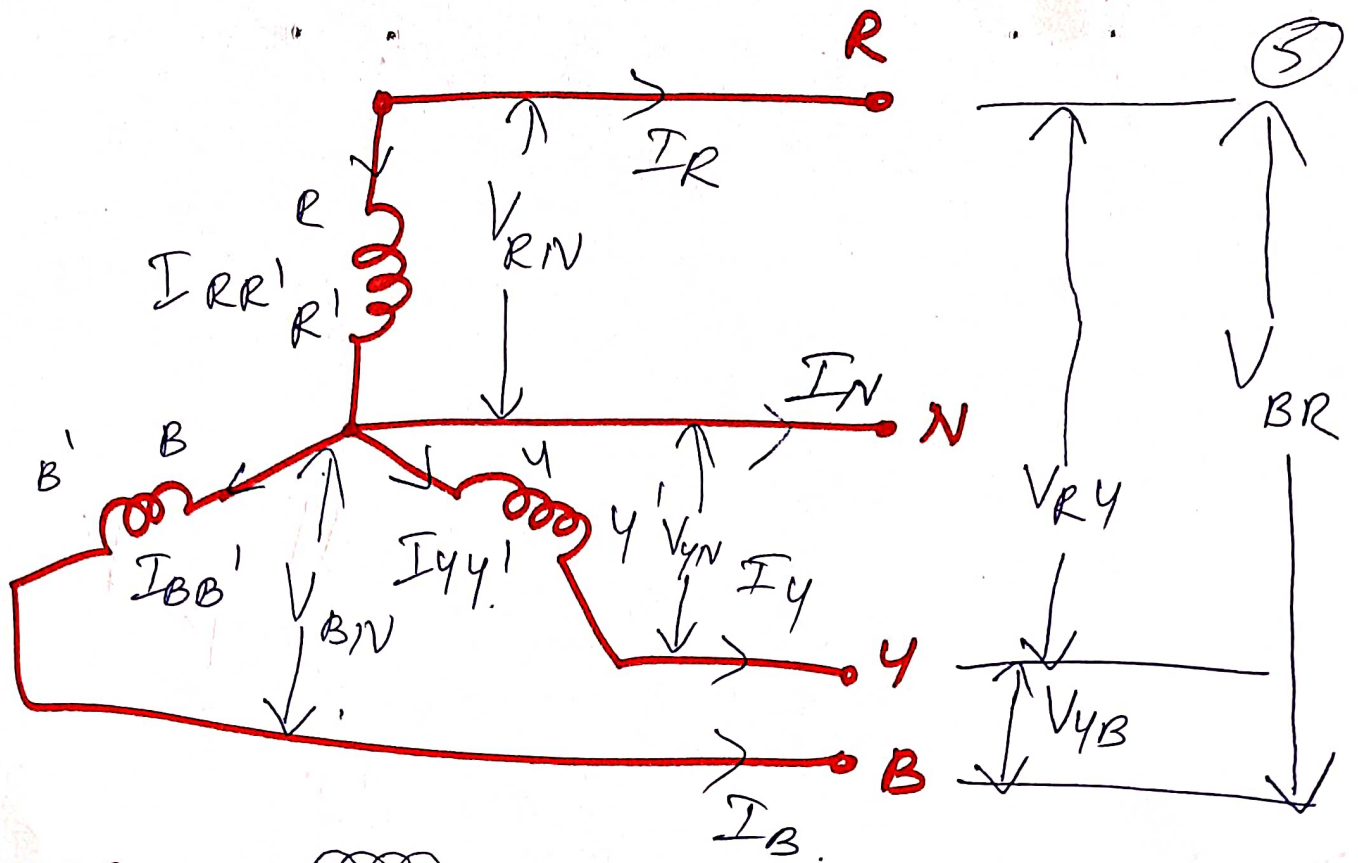
positive phase sequence

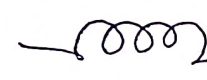


$$V_{RR} = E_{RR} = V_m \sin \omega t$$

$$V_{BB} = E_{BB} = V_m \sin(\omega t - 120^\circ)$$

$$V_{YY} = E_{YY} = V_m \sin(\omega t - 240^\circ)$$



phase - 

line - 

R, Y, B - line terminals

N - Neutral - junction between
R, Y, B phase

Line voltage

Voltage between lines
(R, Y, B lines)

V_{RY} , V_{YB} , V_{BR}

5

Line Current

Current thro' line R, Y, B

I_R, I_Y, I_B

Phase

Between R phase & Neutral N

Y phase & N

B phase & N

Phase voltage

Voltage between phase and Neutral

V_{RN}, V_{YN}, V_{BN}

phase current

Current flowing phase 

I_{RR}, I_{YY}, I_{BB}

$I_{RR'}, I_{YY'}, I_{BB'}$

Neutral current

$I_N \Rightarrow$ current flowing thro' neutral

Basic concept

7

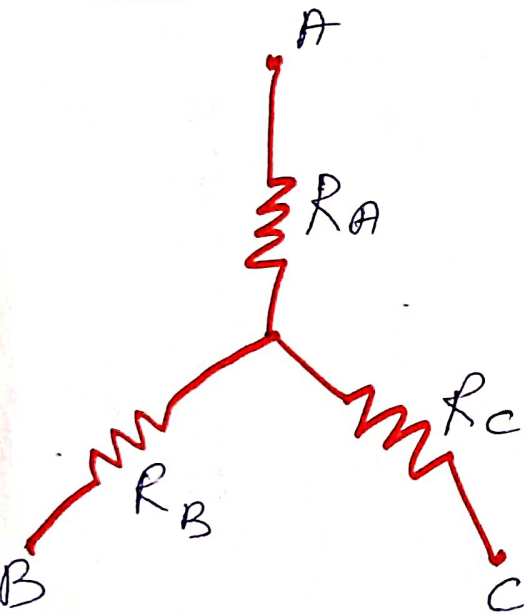
1. Polar to Rectangular form

Rectangular to Polar form.

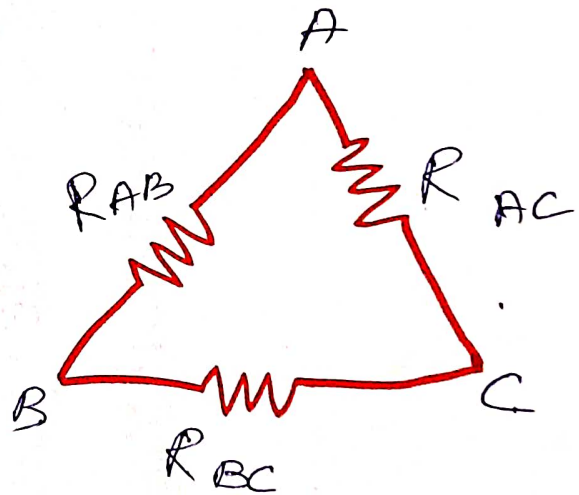
$(a+ib)$ $(r \angle \theta)$

2. star to delta transformation
Delta to star transformation.

Star Network



Delta Network

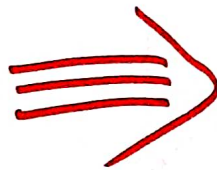
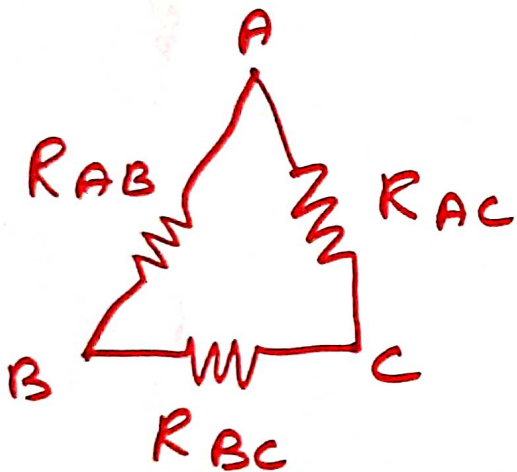


$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

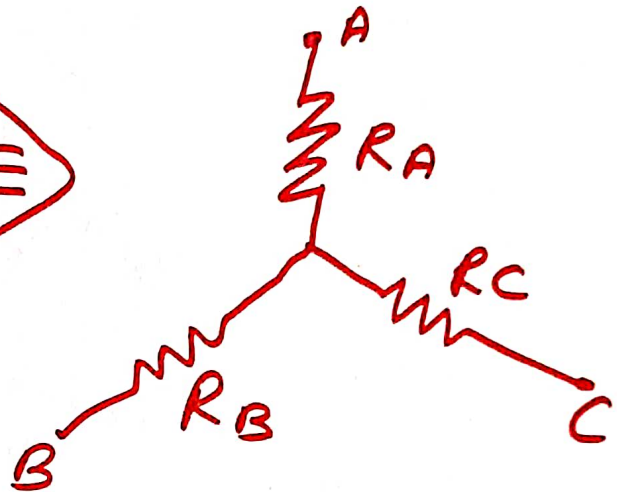
$$R_{AC} = \frac{R_A R_C + R_C R_B + R_B R_A}{R_B}$$

$$R_{BC} = \frac{R_B R_C + R_C R_A + R_A R_B}{R_A}$$

Delta Network



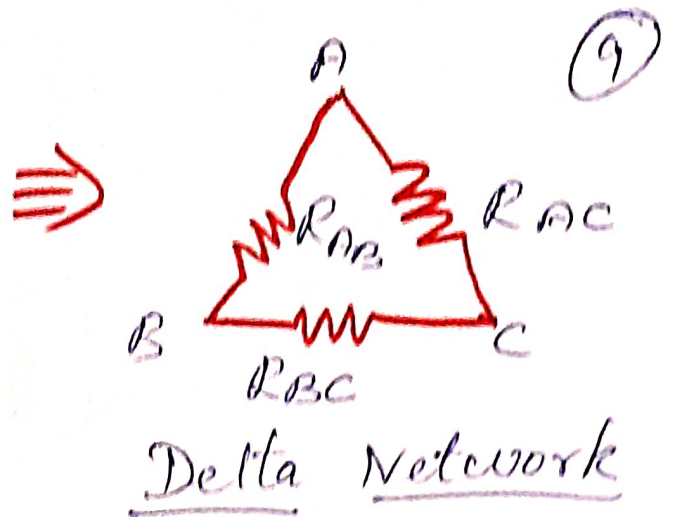
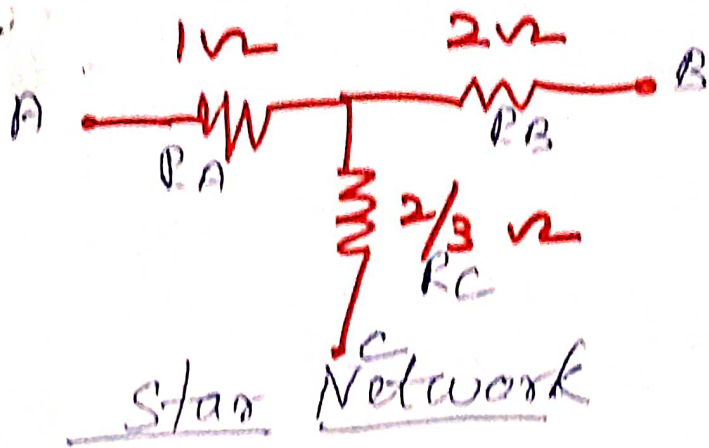
Star Network



$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{BC} + R_{AC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{AC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{BC} + R_{AC}}$$

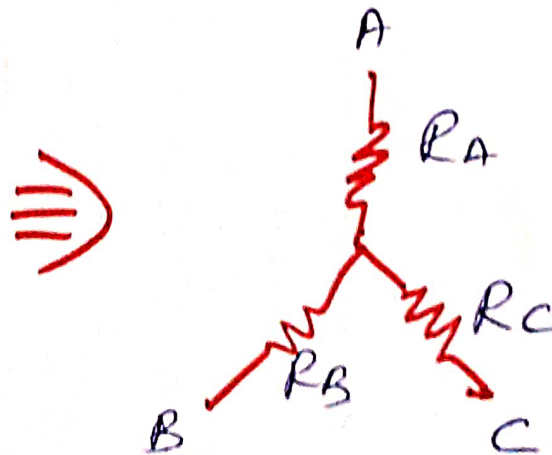
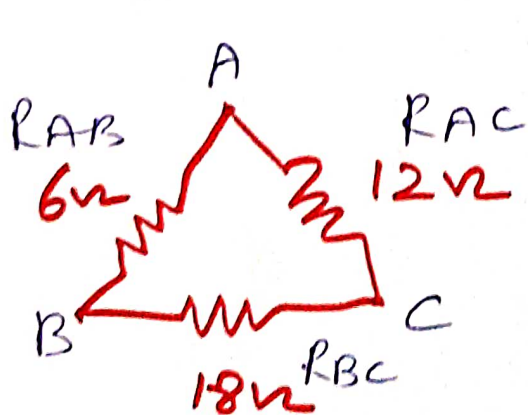


$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$= 6\Omega$$

$$R_{AC} = \frac{N_T}{R_B} = 4\Omega$$

$$R_{BC} = \frac{N_T}{R_A} = 2\Omega$$



$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{BC} + R_{AC}} = 2\Omega$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{AC}} = 6\Omega$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{BC} + R_{AC}} = 3\Omega$$

Balanced system

all R and Z have same values

Unbalanced system

R and Z have different values.

No neutral (N) wire

R, Y, B wire only (11)

A symmetrical 3 ϕ , 3 wire, 400V supply is connected to a delta connected load as shown in figure. Impedance in each branch are

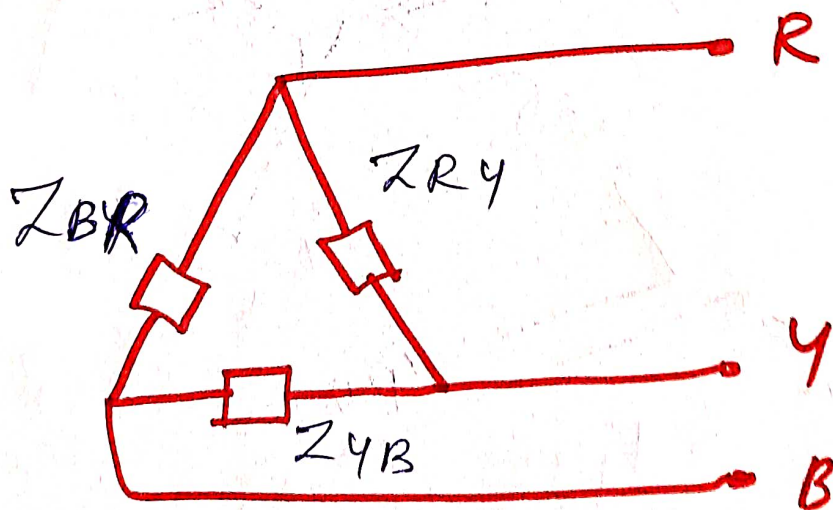
$$Z_{RY} = 10 \angle 30^\circ \Omega$$

$$Z_{YB} = 10 \angle -45^\circ \Omega$$

$$Z_{BR} = 2.5 \angle 60^\circ \Omega$$

Find its equivalent star-connected load. The phase sequence is

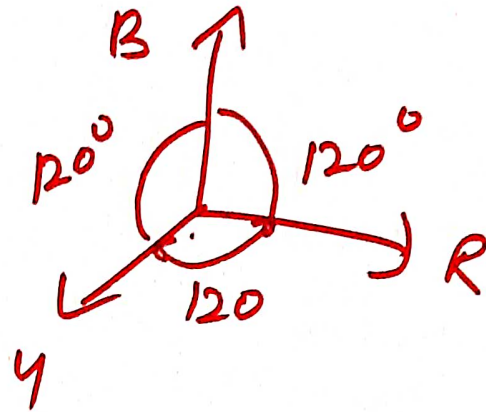
R Y B.



g/p voltage = $V_m \sin \omega t \Rightarrow$ for a.c circuit

(12)

AC circuit three phase



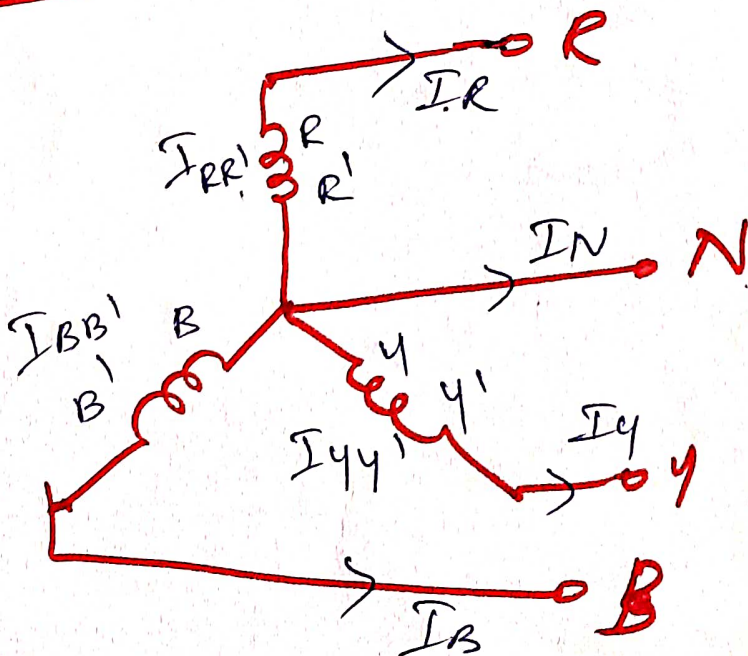
$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin (\omega t - 120^\circ)$$

$$V_B = V_m \sin (\omega t - 240^\circ)$$

$$V_R = E_R, E_m = V_m$$

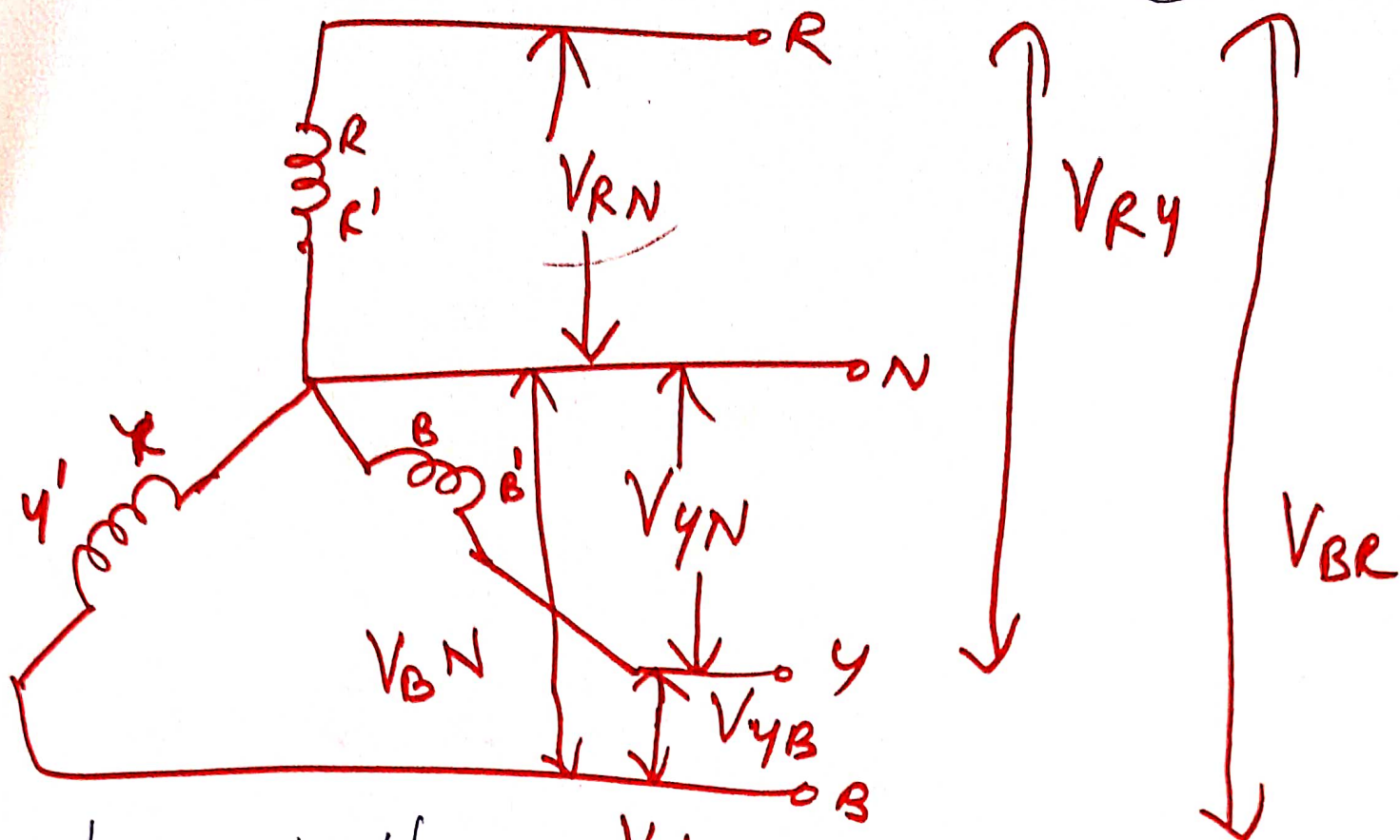
Star connection



Line current: I_L
 I_R, I_Y, I_B

I_N - Neutral current.

Phase current: I_{ph}
 $I_{RR'}, I_{YY'}, I_{BB'}$



Phase voltage : V_{ph}

V_{RN}, V_{YN}, V_{BN}
Between Phase R, Y, B and Neutral N.

Line voltage : V_L
Voltage between lines (R, Y, B)

V_{RY}, V_{YB}, V_{BR}

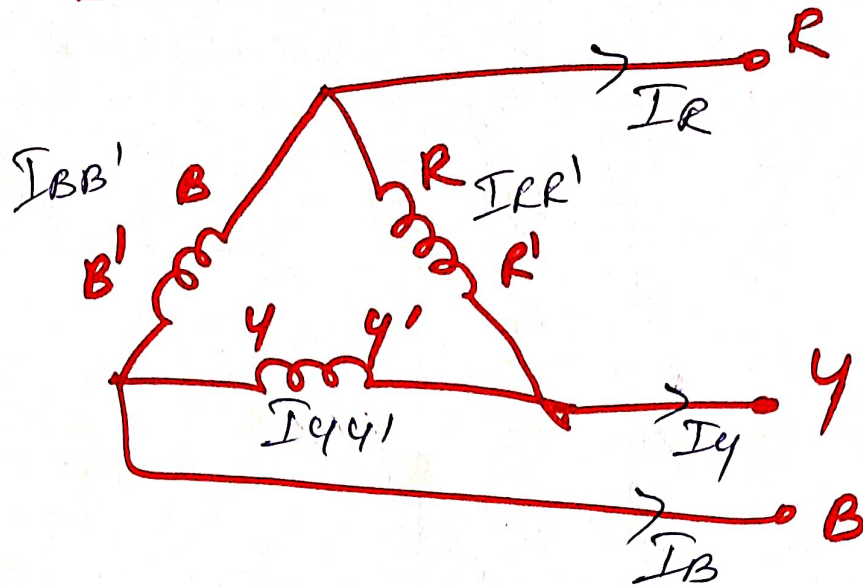
$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

Delta Connection

14

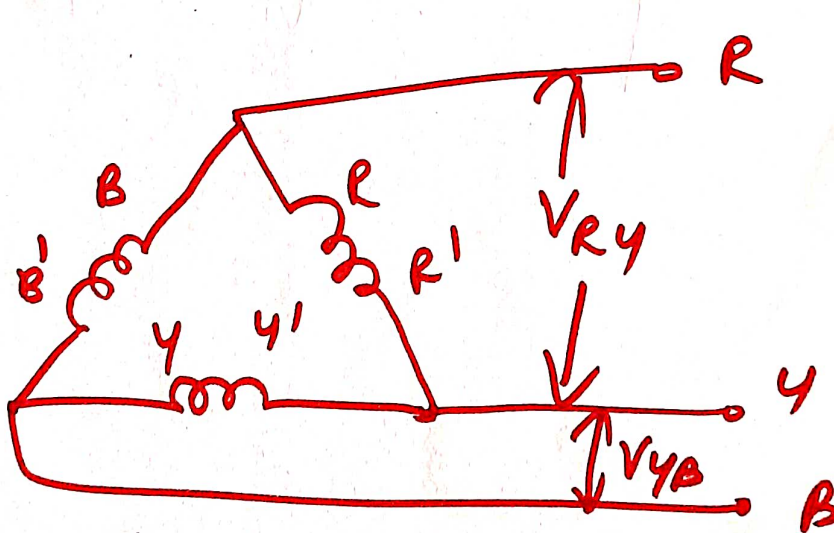


Line current: I_L

I_R, I_Y, I_B

Phase current: I_{ph}

$I_{RR'}, I_{YY'}, I_{BB'}$



phase voltage: $V_{ph} = V_L$

V_{RY}, V_{YB}, V_{BR}

Line voltage: V_L

V_{RY}, V_{YB}, V_{BR}

$$I_L = \sqrt{3} I_{ph}$$

$$V_L = V_{ph}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

A star connected load (balanced) of $(8+j6)\Omega$ per phase is connected to a 3ϕ , $230V$ supply.

Find line current, power factor, power, reactive power and total volt-Amp
star connected load

$$I_L = I_{ph}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$V_L = \sqrt{3} V_{ph}$$

Balanced load, all Z are equal

$$Z = 8+j6\Omega \quad (\text{all impedance - R, Y, B load are same}).$$

$$230V = V_L$$

$$V_L = \sqrt{3} V_{ph}$$

$$\therefore V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.791V$$

$$= 132.791 \angle 0^\circ V$$

$Z = \text{Impedance in each phase.}$
 $= Z_{ph}$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.791 \angle 0^\circ}{8 + j6}$$

16

$$8 + j6 = r \cos \theta$$

$$r = \sqrt{8^2 + 6^2} = 10$$

$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$$

$$I_{ph} = \frac{132.791 \angle 0^\circ}{10 \angle 36.87^\circ}$$

$$= 13.28 \angle 0^\circ - 36.87^\circ$$

$$= 13.28 \angle -36.87^\circ \text{ A.}$$

$$I_{ph} = I_L = 13.28 \angle -36.87^\circ \text{ A.}$$

$$\text{Power factor} = \cos \phi \quad \begin{matrix} \text{negative} \Rightarrow \text{lagging} \\ (\phi) \end{matrix}$$

$$\phi = -36.87^\circ$$

$$\text{Power factor} = \cos(36.87^\circ)$$

$$= 0.7999 = 0.8$$

Power factor must be less than one

P = Active Power (W)

$$\text{Power} = \sqrt{3} V_L I_L \cos \phi \text{ W}$$

$$= \sqrt{3} \times 230 \times 13.28 \angle -36.87^\circ \times 0.8$$

$$= 4232.30 \angle -36.87^\circ \text{ Watt}$$

$$= 4.232 \angle -36.87^\circ \text{ kilo Watt.}$$

Reactive Power (Q)

$$\sqrt{3} V_L I_L \sin \phi \text{ VAR}$$

$$= \sqrt{3} \times 230 \times 13.28 \angle -36.87^\circ \times \sin (-36.87^\circ)$$

$$= 5290.38 \angle -36.87^\circ \times -0.6$$

$$= -3174.228 \angle -36.87^\circ \text{ VAR.}$$

Apparent Power (S)

$$\sqrt{3} V_L I_L \text{ VA}$$

$$= \sqrt{3} \times 230 \times 13.28 \angle -36.87^\circ$$

$$= 5290.38 \angle -36.87^\circ \text{ VA}$$

(one phase)

$$\text{Total VA} = 3 \times \text{Apparent power.}$$

$$= 3 \times 5290.38 \angle -36.87.$$

$$= 15871.14 \angle -36.87 \text{ VA.}$$

If needed to calculate power in phase use V_{ph} , I_{ph} .

If "phase" is specified in problem.

A balanced delta-connected load of $(2+j3)\Omega$ per phase is connected to a balanced 3 ϕ 440V supply. The phase current is 10A. Find

- i) total active power
- ii) ~~tot~~ Reactive power
- iii) apparent power.

Delta connected load

$$I_L = \sqrt{3} I_{ph}$$

$$V_L = V_{ph}$$

$$V_L = 440 \text{ V}, V_{ph} = 440 \text{ V}$$

$$Z = (2 + j3) \Omega$$

19

$$I_{ph} = 10 A \Rightarrow I_L = \sqrt{3} I_{ph}$$

$$= 17.32 A$$

To calculate power we need power factor, ϕ

$$\text{power factor } \cos \phi = \frac{R_{ph}}{Z_{ph}}$$

$$\phi = \cos^{-1} \left(\frac{R_{ph}}{Z_{ph}} \right)$$

$$Z = (2 + j3) \Omega = (R + j) \Omega$$

$$R = 2 \Rightarrow R_{ph}$$

$$Z = 2 + j3$$

$$r = \sqrt{2^2 + 3^2} = 3.6$$

$$\theta = \tan^{-1}(b/a) = \tan^{-1}(3/2) = 56.31$$

$$Z = 3.6 \angle 56.31$$

$$\cos \phi = \frac{R_{ph}}{Z_{ph}} = \frac{2}{3.6} = 0.56$$

$$\phi = \cos^{-1}(0.56) = 55.94$$

Active Power / Total Active Power

20

$$P = \sqrt{3} V_L I_L \cos \phi \text{ W}$$

$$= 1.732 \times 440 \times 17.32 \times 0.56$$

$$= 7391.57 = 7.39157 \text{ kW}$$

Reactive Power

$$Q = \sqrt{3} V_L I_L \sin \phi \text{ VAR}$$

$$= 1.732 \times 440 \times 17.32 \times$$

$$\sin (55.94) = 10934.92$$

$$= 10.9349 \text{ kVAR}$$

Apparent Power

$$(S) = \sqrt{3} V_L I_L \text{ VA}$$

$$= 1.732 \times 440 \times 17.32$$

$$= 13199.23 \text{ VA}$$

$$= 13.199 \text{ kVA}$$

(21)

A 3 ϕ , balanced delta-connected load of $(4 + j8) \Omega$ is connected across 400 V, 3 ϕ balanced supply. Determine phase currents and line currents. Assume RYB phase sequence. Calculate power drawn by the load.

$$V_L = V_{ph} \quad , \quad I_L = \sqrt{3} I_{ph}$$

$$V_L = 400 \text{ V}$$

$$Z_{ph} = (4 + j8) \Omega$$

$$= 8.94 \angle 63.43$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

Phase voltages

$$V_{RR'} = 400 \angle 0^\circ \text{ V} = \text{Phase voltage in "R"}$$

$$V_{YY'} = 400 \angle -120^\circ \text{ V} = \dots \text{ "Y"}$$

$$V_{BB'} = 400 \angle -240^\circ \text{ V} = \dots \text{ "B"}$$

Phase currents

$$I_{RR'} = \frac{V_{RR'}}{Z_{ph}} = \frac{400 \angle 0^\circ}{8.94 \angle 63.43^\circ}$$

$$= 44.74 \angle -63.43^\circ \text{ A}$$

$$I_{YY'} = \frac{V_{YY'}}{Z_{ph}} = \frac{400 \angle -120^\circ}{8.94 \angle 63.43^\circ}$$

$$= 44.74 \angle -183.43^\circ \text{ A}$$

$$I_{BB'} = \frac{V_{BB'}}{Z_{ph}} = \frac{400 \angle -240^\circ}{8.94 \angle 63.43^\circ}$$

$$= 44.74 \angle -303.43^\circ \text{ A.}$$

Line currents

$$I_L = \sqrt{3} I_{ph}$$

$$I_R = \sqrt{3} I_{RR'} = 1.732 \times 44.74$$

$$\angle -63.43^\circ = 77.49 \angle -63.43^\circ \text{ A}$$

$$I_Y = \sqrt{3} I_{YY'} = 1.732 \times 44.74 \angle -183.43^\circ$$

$$= 77.49 \angle -183.43^\circ \text{ A}$$

$$I_B = \sqrt{3} I_{BB'} = 1.732 \times 44.74 \angle -303.43^\circ$$

$$= 77.49 \angle -303.43^\circ \text{ A}$$

Power drawn by the load = Active power
 $P = \sqrt{3} V_L I_L \cos \phi$ = Total power.

$$V_L = 400 \text{ V}$$

$$I_L = 77.49$$

$$\phi = 63.43^\circ$$

$$Z_{ph} = 8.94 \angle 63.43^\circ$$

$\text{---} \phi$

$$P = 1.732 \times 400 \times 77.49 \times \cos 63.43$$

$$= 24012.8414 \text{ W}$$

$$= 24.013 \text{ kW}$$

$$\text{Reactive Power } Q = \sqrt{3} V_L I_L \sin \phi \text{ VAR.}$$

$$= 48015.31426 \text{ VAR}$$

$$= 48.015 \text{ kVAR}$$

$$\text{Apparent Power } S = \sqrt{3} V_L I_L \text{ VA}$$

$$= 53685.072 \text{ VA}$$

$$= 53.685 \text{ kVA}$$

A balanced star-connected load having an impedance $(15 + j20)\Omega$ per phase is connected to a 3 ϕ , 440 V, 50 Hz supply. Find line currents and power absorbed by the load. RYB phase sequence. 24

$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

$$Z = (15 + j20)\Omega = 25 \angle \underline{53.13}^\circ \Omega$$

$$V_L = 440 \text{ V.}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

phase voltage

To calculate phase current in each phase, calculate phase voltage in each phase.

$$V_{RR'} = \frac{440 \angle 0^\circ}{\sqrt{3}} = 254.04 \angle 0^\circ \text{ V}$$

$$V_{YY'} = \frac{440 \angle -120^\circ}{\sqrt{3}} = 254.04 \angle -120^\circ \text{ V}$$

$$V_{BB'} = \frac{440 \angle -240^\circ}{\sqrt{3}} = 254.04 \angle -240^\circ \text{ V}$$

phase current

25

$$I_{RR'} = \frac{V_{RR'}}{Z} = \frac{254.04 \angle 0^\circ}{25 \angle 53.13^\circ} = 10.16 \angle -53.13^\circ \text{ A}$$

$$\begin{aligned} I_{YY'} &= \frac{V_{YY'}}{Z} = \frac{254.04 \angle -120^\circ}{25 \angle 53.13^\circ} \\ &= 10.16 \angle -173.13^\circ \text{ A} = 10.16 \angle -173.13^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} I_{BB'} &= \frac{V_{BB'}}{Z} = \frac{254.04 \angle -240^\circ}{25 \angle 53.13^\circ} \\ &= 10.16 \angle -293.13^\circ \text{ A} \end{aligned}$$

Line current = phase current

$$I_R = I_{RR'}$$

$$I_Y = I_{YY'}$$

$$I_B = I_{BB'}$$

Power absorbed by the load

$$\begin{aligned} P &= \sqrt{3} V_L I_L \cos \phi = 1.732 \times 440 \times \\ &\quad 10.16 \cos (53.13^\circ) \\ &= 4645.65 \text{ W} = 4.646 \text{ kW} \end{aligned}$$

Balanced load = load in each phase (impedance = Z) are equal (26)

Unbalanced load = load is different in each phase.

For power calculation, calculate power in each phase separately and then add power in three phases together.

phase current and phase voltage are different for each phase
line voltage and line current are different for each phase.

Three impedances $Z_1 = 20 \angle 30^\circ \Omega$,
 $Z_2 = 40 \angle 60^\circ \Omega$, $Z_3 = 10 \angle -90^\circ \Omega$
 (unbalanced load) are connected
 to a delta supply of 400V, 3 ϕ
 system. Determine phase currents,
 line currents, total power
 consumed by the load.

$$I_L = \sqrt{3} I_{ph} \quad , \quad V_L = V_{ph}$$

$$V_L = 400 \text{ V.}$$

phase voltage.

$$V_{RR'} = 400 \angle 0^\circ \text{ V}$$

$$V_{YY'} = 400 \angle -120^\circ \text{ V}$$

$$V_{BB'} = 400 \angle -240^\circ \text{ V}$$

$$Z_1 = Z_R, \quad Z_2 = Z_Y, \quad Z_3 = Z_B.$$

Phase currents

(28)

$$I_{RR'} = \frac{V_{RR'}}{Z_R} = \frac{400 \angle 0^\circ}{20 \angle 30^\circ} = 20 \angle -30^\circ \text{ A}$$

$$I_{YY'} = \frac{V_{YY'}}{Z_Y} = \frac{400 \angle -120^\circ}{40 \angle 60^\circ} = 10 \angle -180^\circ \text{ A}$$

$$I_{BB'} = \frac{V_{BB'}}{Z_B} = \frac{400 \angle -240^\circ}{10 \angle -90^\circ} = 40 \angle -150^\circ \text{ A}$$

Line currents

$$I_L = \sqrt{3} I_{ph.}$$

$$I_R = \sqrt{3} \times I_{RR'} = 1.732 \times 20 \angle -30^\circ \\ = 34.64 \angle -30^\circ \text{ A}$$

$$I_Y = \sqrt{3} \times I_{YY'} = 1.732 \times 10 \angle -180^\circ \\ = 17.32 \angle -180^\circ \text{ A}$$

$$I_B = \sqrt{3} \times I_{BB'} = 1.732 \times 40 \angle -150^\circ \\ = 69.28 \angle -150^\circ \text{ A}$$

(29)

Power in R phase

$$= \sqrt{3} V_{ph} I_{ph}$$

$$= \sqrt{3} V_{RR'} I_{RR'} = 13856 \angle -30^\circ$$

Power in Y phase

$$= \sqrt{3} V_{YY'} I_{YY'} = 6928 \angle -30^\circ$$

Power in B phase

$$= \sqrt{3} V_{BB'} I_{BB'} = 27712 \angle -30^\circ$$

Total power = Sum of power in
R, Y, B phases.

$$= 13856 + 6928 + 27712 = 48496 \text{ W}$$

$$= 48.496 \text{ kW}$$

(omit angle part)

Neutral shift

Neutral point - junction of R, Y, B
connect wire to a neutral point
→ Neutral line (N)

Current thro' Neutral line is
Neutral current I_N

$$I_N = - [I_R + I_Y + I_B]$$

Neutral shift occurs due to
unbalanced load.

$$V_N = (V_{RY} \times V_{RN}) + (V_{YB} \times V_{YN}) \\ + (V_{BR} \times V_{BN})$$

$$V_{RY} + V_{YB} + V_{BR}$$

= Sum of product of line voltage
and phase voltage

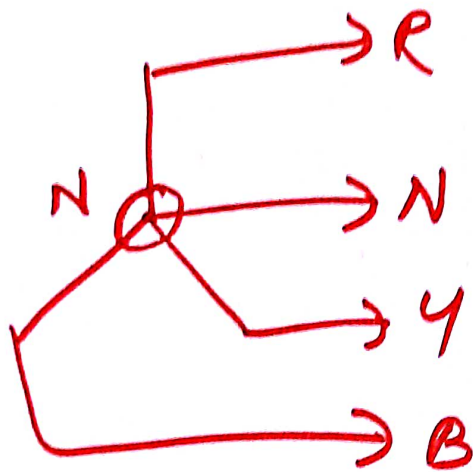
Sum of line voltage.

Unbalanced Four Wire Star Connected Load

Four wire $\rightarrow$ R, Y, B and
Neutral (N)

Star - Y Network.

Delta - Δ Network.



Neutral point
available in
star Network.

Star Network, four wires possible
Wire means line, R, Y, B, N

$$I_N = - [I_R + I_Y + I_B]$$

An unbalanced four-wire, star connected load has a balanced voltage of 400V. The loads are

$$Z_1 = (4 + j8) \Omega = Z_R$$

$$Z_2 = (3 + j4) \Omega = Z_Y$$

$$Z_3 = (15 + j20) \Omega = Z_B$$

Calculate line currents, current in neutral line, total power.

$$V_L = 400V$$

$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} \Rightarrow \text{Phase voltage}$$

$$V_{RR'} = \frac{400}{1.732} \angle 0^\circ$$

$$V_{YY'} = \frac{400}{1.732} \angle -120^\circ$$

$$V_{BB'} = \frac{400}{1.732} \angle -240^\circ$$

$$Z_R = (4 + j8) \Omega =$$

$$Z_Y = (3 + j4) \Omega =$$

$$Z_B = (15 + j20) \Omega$$

phase current

$$I_{RR'} = \frac{V_{RR'}}{Z_R} =$$

$$I_{YY'} = \frac{V_{YY'}}{Z_Y} =$$

$$I_{BB'} = \frac{V_{BB'}}{Z_B} =$$

$$I_L = I_{ph}$$

$$I_{RR'} = I_R$$

$$I_{YY'} = I_Y$$

$$I_{BB'} = I_B$$

Neutral Line current I_N

$$= - [I_R + I_Y + I_B]$$

Convert I_R, I_Y, I_B in rectangular form for addition

$$\text{Power in R phase} = \sqrt{3} V_L I_L \cos \phi$$

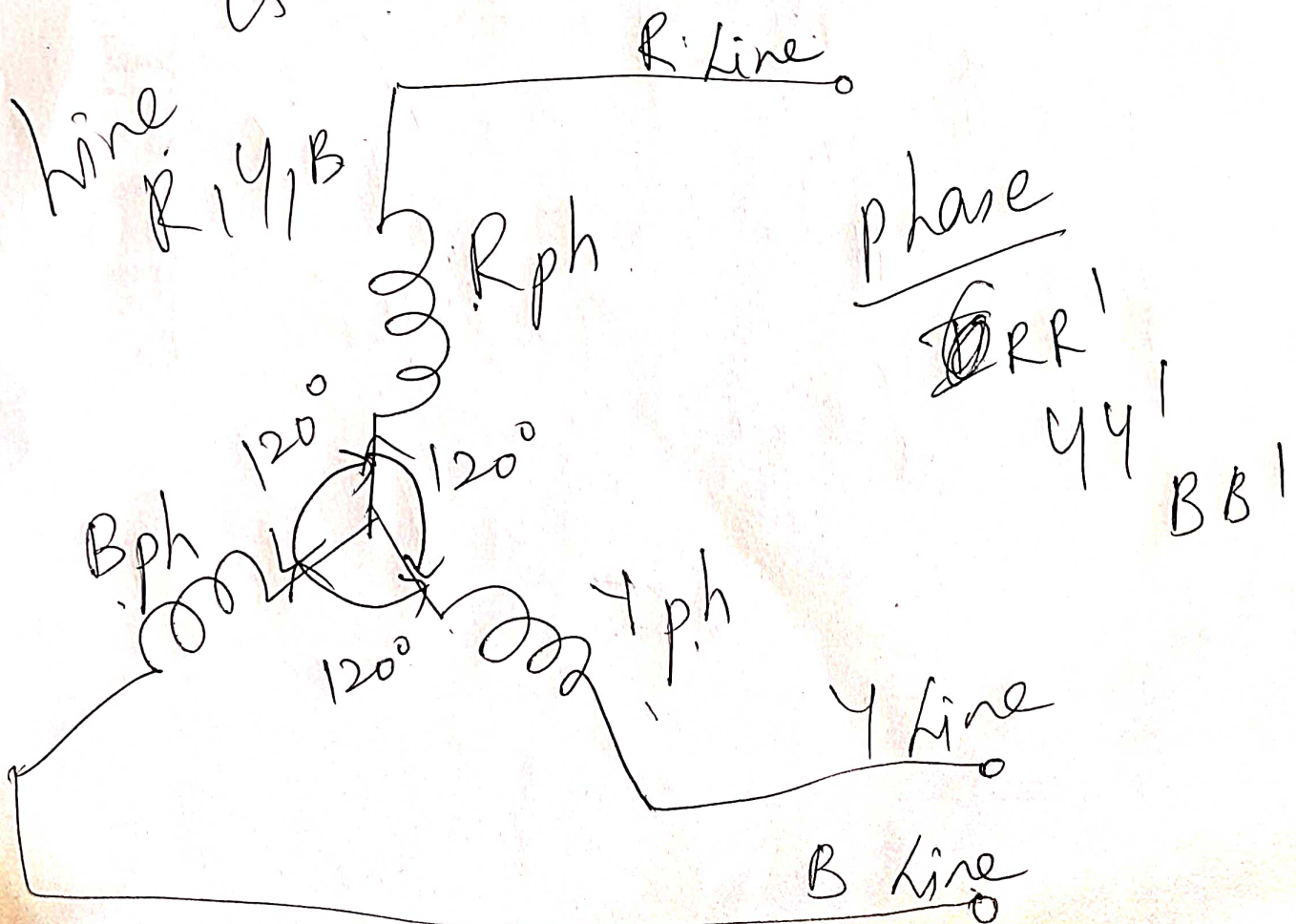
$$= \sqrt{3} V_{RR'} I_{RR'} \cos \phi$$

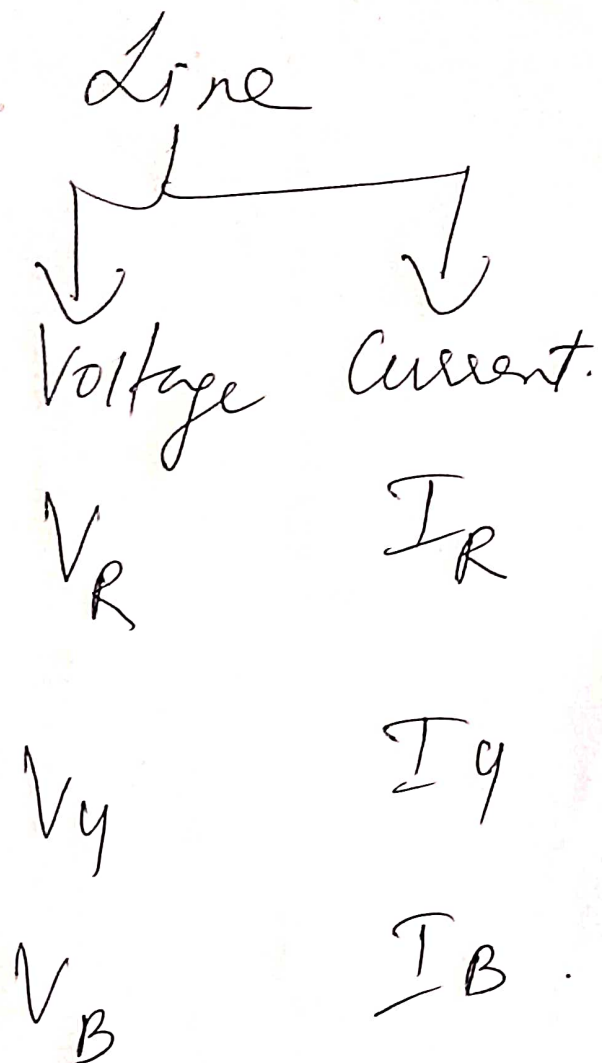
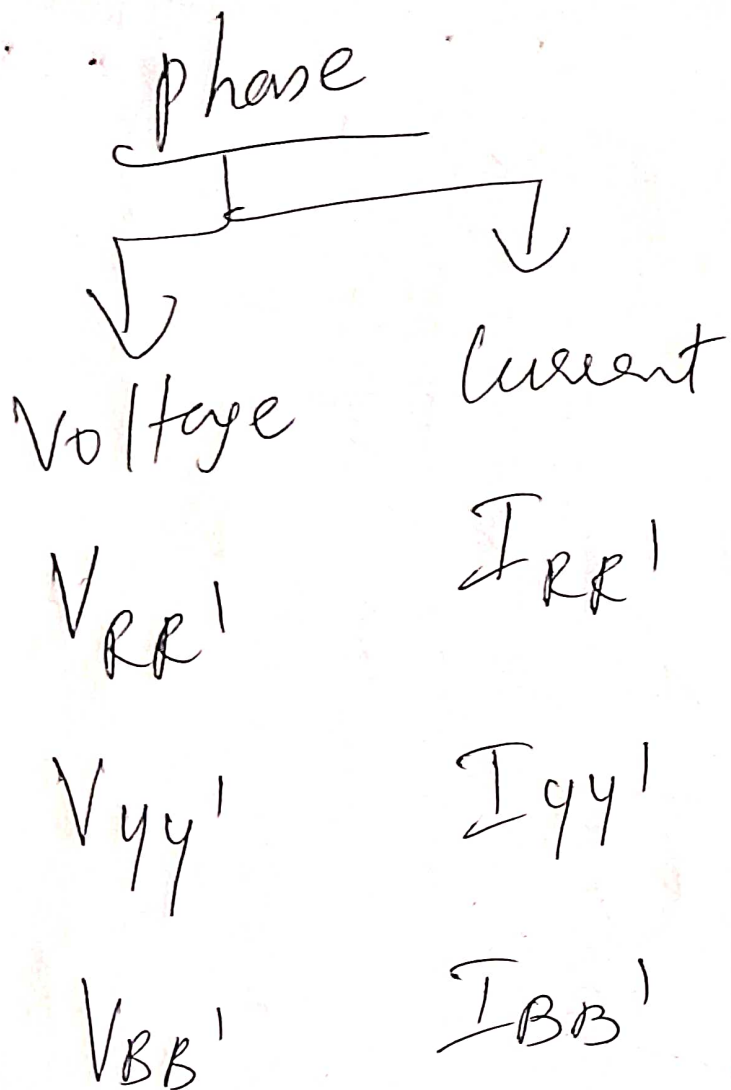
ϕ from ^{angle} ~~R~~ in Z_R (polar form)

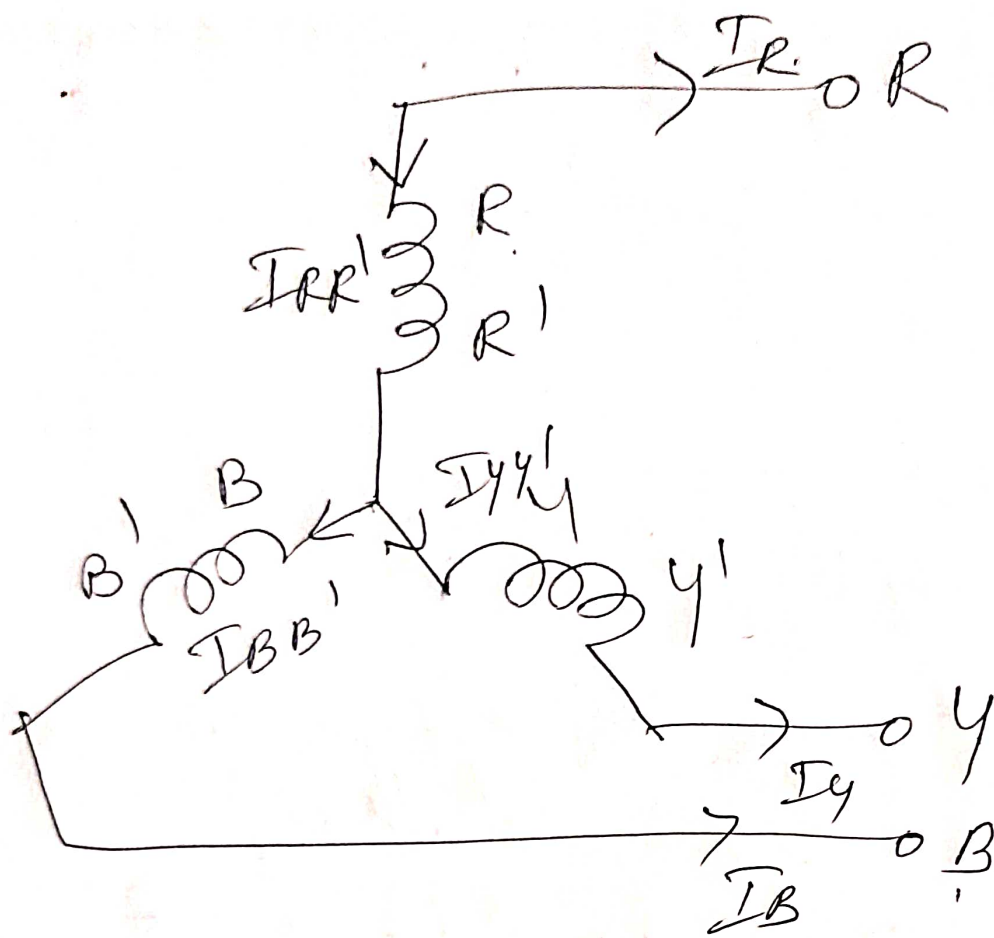
— phase.



R, Y, B phase connected
With wire — This wire
is line.







Phase current

Current flowing thro' phase

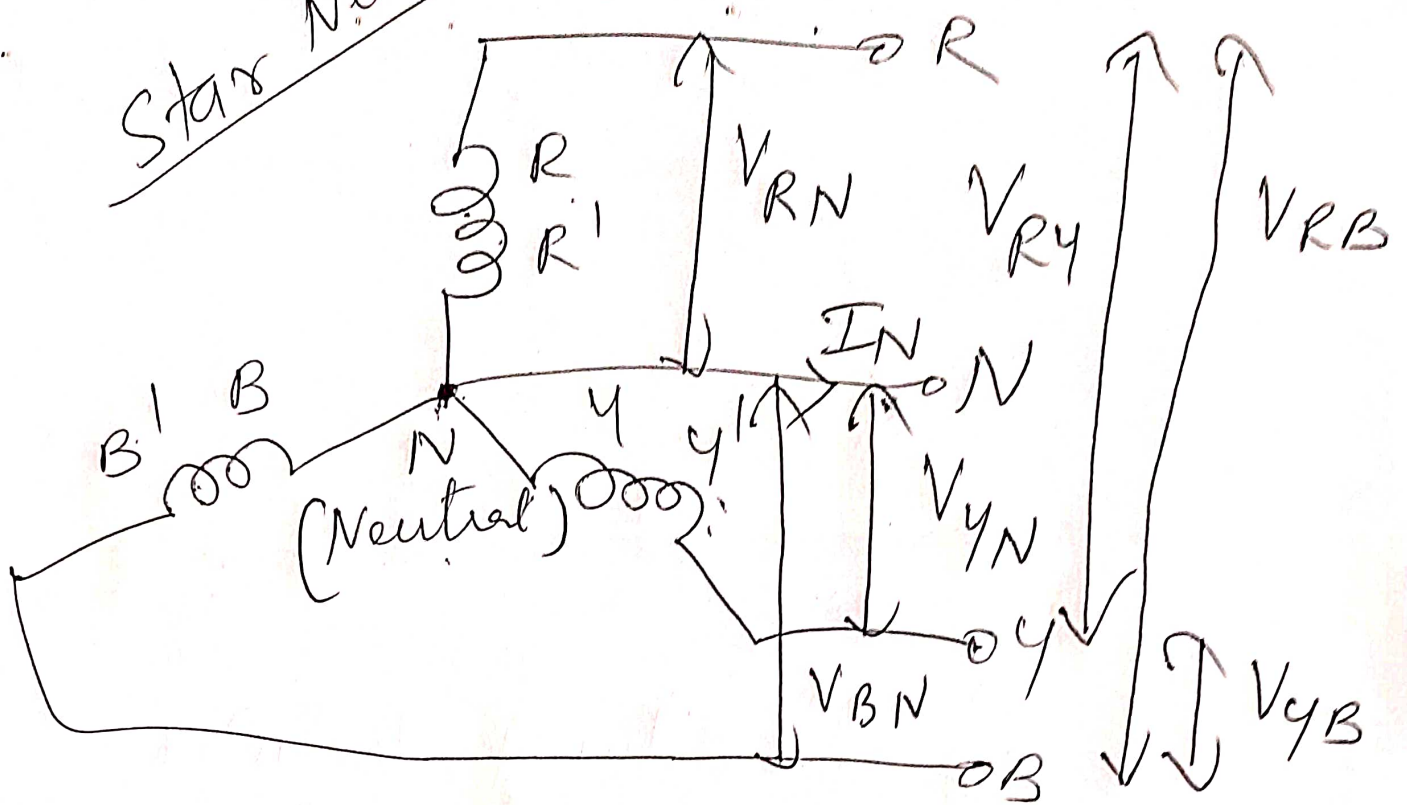
$I_{RR'}$, $I_{YY'}$, $I_{BB'}$

Line current

Current flowing thro' line

I_R , I_Y , I_B .

Star Network



Line Voltage

Voltage Between line

$$V_{RY}, V_{YB}, V_{BR}.$$

phase Voltage

Voltage between phase and Neutral.

$$V_{RN}, V_{YN}, V_{BN}$$

Neutral point

Junction at which, R, Y, B phase join together.

Connecting wire to neutral point

↳ Neutral line

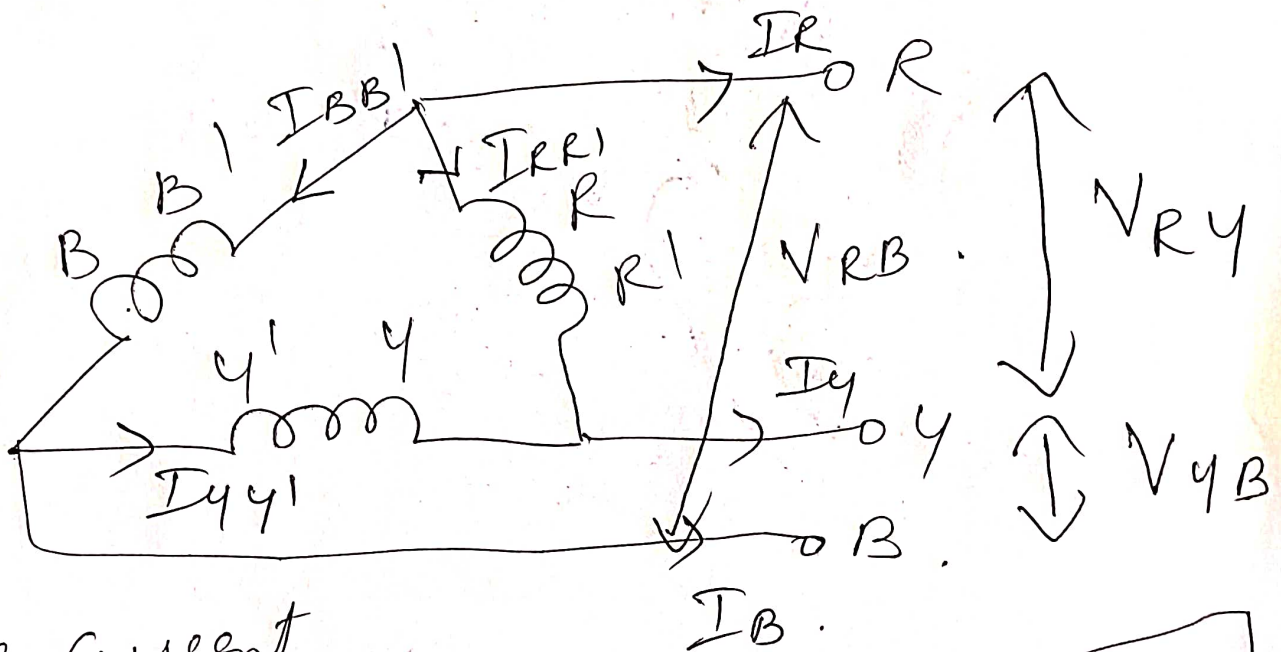
↳ current flowing thro'

Neutral line is Neutral current.

(IN)

Only in star network,
Neutral point, Neutral
current.

Delta Network



Phase current

$I_{BB'}$, $I_{RR'}$, $I_{YY'}$

Line current

I_R , I_Y , I_B .

No neutral point
in delta
Network

Line voltage

Voltage between line

V_{RY} , V_{YB} , V_{RB} .

[No neutral]

$$V_{\text{phase}} = V_{\text{line}}$$

Module 5

①

Z parameters

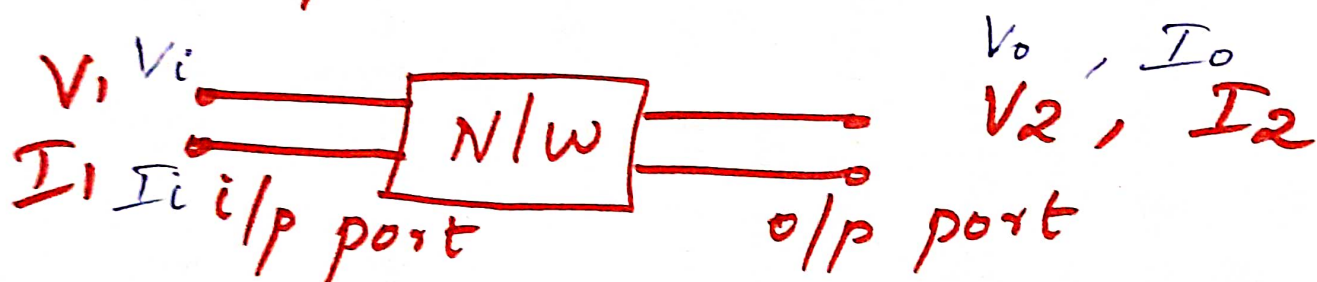
$$\text{Impedance} = Z = \frac{V}{I}$$

Impedance parameters

open circuit parameters

open circuit impedance parameters.

Two port network.



For a two port N/w, the i/p and o/p voltages V_1 and V_2 can be represented / expressed in terms of i/p and o/p current I_1 and I_2 .

$$[V] = [Z] [I]$$

$$V = RI = IR$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

From (1) and (2), find Z_{11} , Z_{12} , Z_{21} , Z_{22}

put $I_2 = 0$, open circuit I_2 .
o/p port open

$$V_1 = Z_{11} I_1$$

$$Z_{11} = \frac{V_1}{I_1} \Rightarrow \text{I/p driving point impedance}$$

$$V_2 = Z_{21} I_1$$

$$Z_{21} = \frac{V_2}{I_1} \Rightarrow \text{Forward Transfer Impedance}$$

put $I_1 = 0$, open circuit I_1

$$V_1 = Z_{12} I_2$$

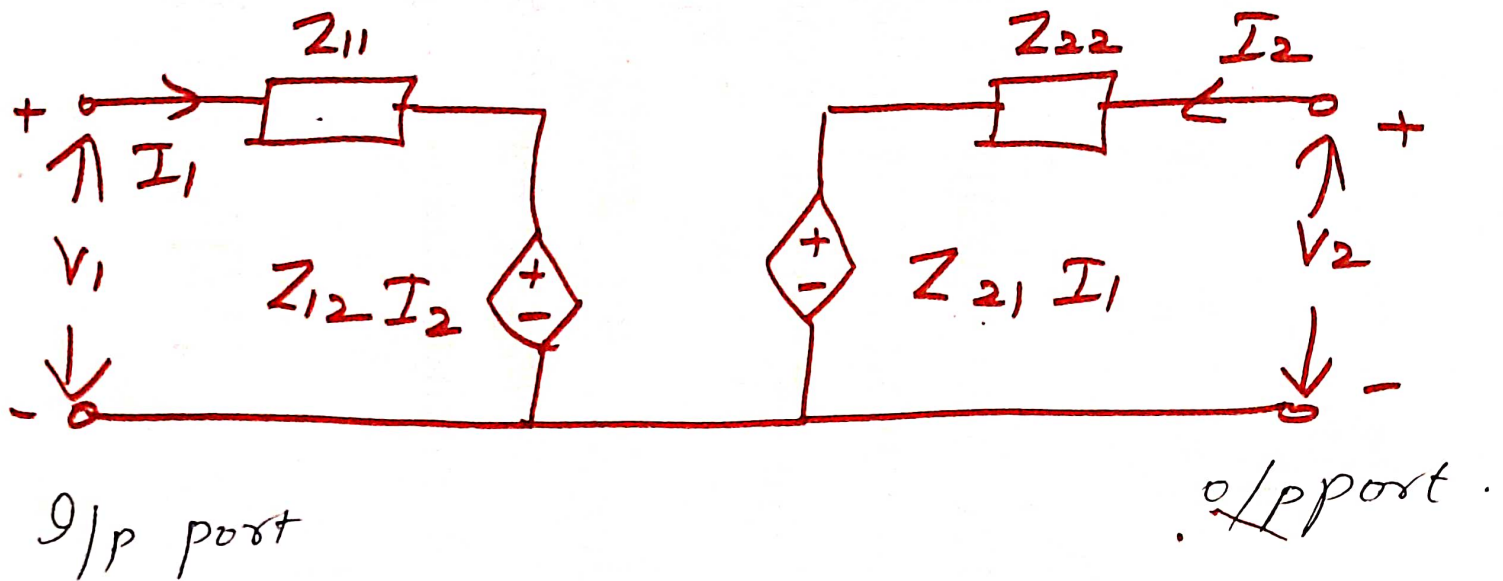
$$V_2 = Z_{22} I_2$$

$$Z_{12} = \frac{V_1}{I_2}$$

$$Z_{22} = \frac{V_2}{I_2}$$

Reverse Transfer Impedance $\downarrow$ o/p, driving point impedance

2 parameter Equivalent circuit (3)



Y parameters

Admittance Parameters

Short circuit Parameters

Short circuit Admittance parameters



$$V = I R$$

$$I = \frac{V}{R} \Rightarrow I = Y V$$

$$[I] = [Y] [V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (11)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

From (1) and (2), find Y parameters.

put $V_2 = 0$, short circuit V_2

$$I_1 = Y_{11} V_1$$

$$I_2 = Y_{21} V_1$$

$$Y_{11} = \frac{I_1}{V_1}$$

$$Y_{21} = \frac{I_2}{V_1}$$

put $V_1 = 0$, short circuit V_1

$$I_1 = Y_{12} V_2$$

$$I_2 = Y_{22} V_2$$

$$Y_{12} = \frac{I_1}{V_2}$$

$$Y_{22} = \frac{I_2}{V_2}$$

$Y_{11} \Rightarrow$ Input driving point admittance

$Y_{22} \Rightarrow$ Output driving point admittance

$Y_{12} \Rightarrow$ Reverse Transfer admittance

$Y_{21} \Rightarrow$ forward Transfer admittance.

h parameters

Hybrid parameters.

Both open ^($I=0$) and short circuit ^($V=0$) conditions.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

write eqn for i/p voltage V_1 and o/p current I_2 .

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Using (1) and (2) find "h" parameters.

$V_2 = 0$, short circuit V_2

$$V_1 = h_{11} I_1$$

$$I_2 = h_{21} I_1$$

$$h_{11} = \frac{V_1}{I_1}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{\text{o/p } I}{\text{i/p } I}$$

put $I_1 = 0$, open circuit I_1

⑥

$$V_1 = h_{12} V_2$$

$$I_2 = h_{22} V_2$$

$$h_{12} = \frac{V_1}{V_2}$$

$$h_{22} = \frac{I_2}{V_2}$$

$h_{11} \Rightarrow$ input impedance

$h_{21} \Rightarrow$ forward current gain

$h_{12} \Rightarrow$ Reverse voltage gain

$h_{22} \Rightarrow$ output admittance.

T parameters

ABCD parameters

Circuit parameters

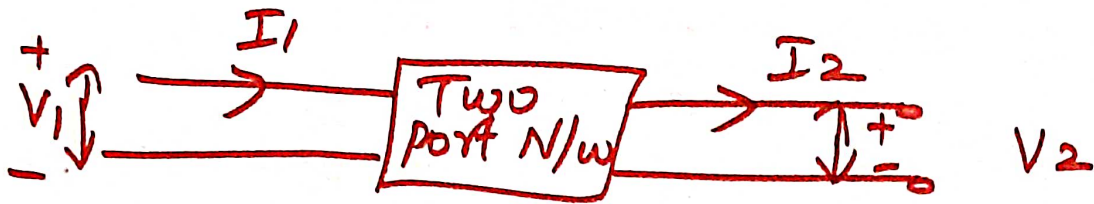
Transmission parameters.

Used in power transmission engineering.

power transmitted from one end (i/p)
sending end / port

power received at other end (o/p)
receiving end / port

(7)



sending end
i/p port

Receiving end
o/p port

direction of I_2 is towards
right side ($\therefore$ receiving)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

Using (1) and (2) find T parameters.

$I_2 = 0$, receiving end is open circuited

$$V_1 = AV_2$$

$$I_1 = CV_2$$

$$A = \frac{V_1}{V_2}$$

$$C = \frac{I_1}{V_2}$$

put $V_2 = 0$, receiving end is short circuited.

(8)

$$V_1 = -B I_2$$

$$I_1 = -D I_2$$

$$B = -\frac{V_1}{I_2}$$

$$D = -\frac{I_1}{I_2}$$

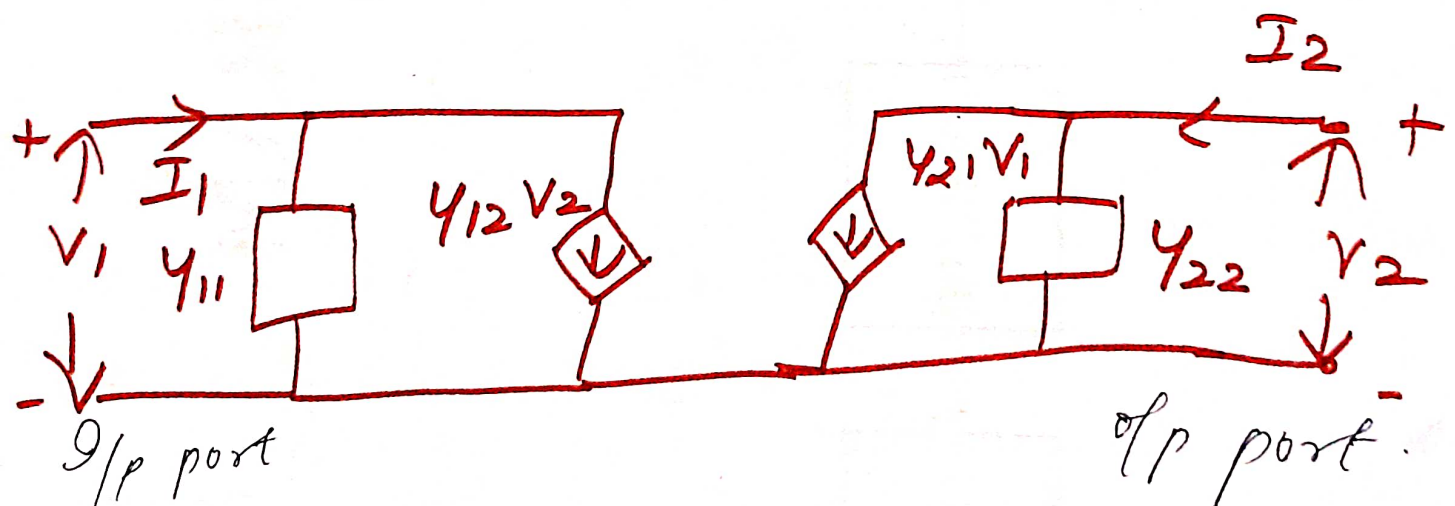
A $\Rightarrow$ Reverse voltage ratio

B $\Rightarrow$ Transfer Impedance

C $\Rightarrow$ Transfer Admittance

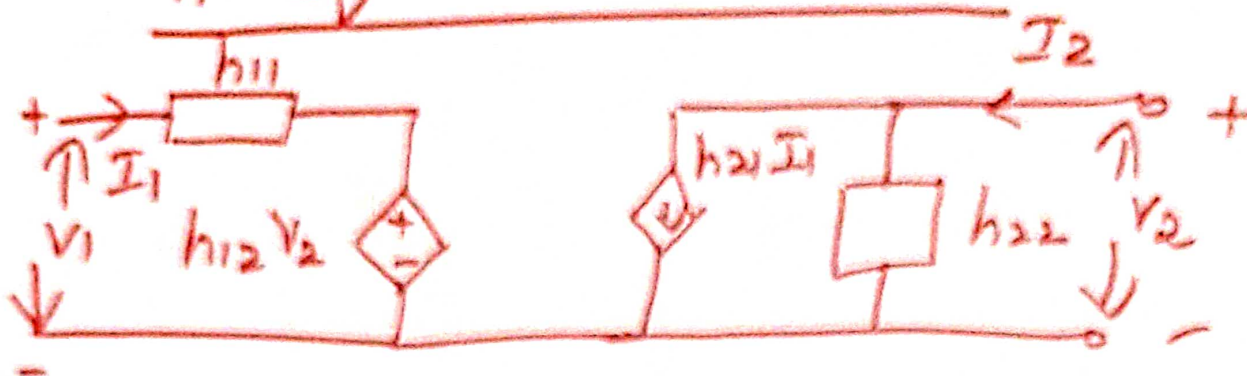
D $\Rightarrow$ Reverse Current Ratio

Y equivalent circuit



h-equivalent circuit

(9)



Inverse Transmission Parameters.

$A' B' C' D'$ parameters.

$$V_2 = A' V_1 - B' I_1$$

$$I_2 = C' V_1 - D' I_1$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ C' & -D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$I_1 = 0$$

$$V_2 = A' V_1$$

$$A' = \frac{V_2}{V_1}$$

$$V_1 = 0$$

$$V_2 = -B' I_1$$

$$B' = -\frac{V_2}{I_1}$$

$$I_2 = C' V_1$$

$$C' = \frac{I_2}{V_1}$$

$$I_2 = -D' I_1$$

$$D' = -\frac{I_2}{I_1}$$

A' = Inverse Forward voltage Ratio

B' = Inverse transfer impedance

C' = Inverse transfer admittance

D' = Inverse forward current gain

Parameter	Conditions for	
	Symmetry	Reciprocity
Z	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
h	$\Delta h = 1$	$h_{12} = -h_{21}$
T	$A = D$	$AD - BC = 1$

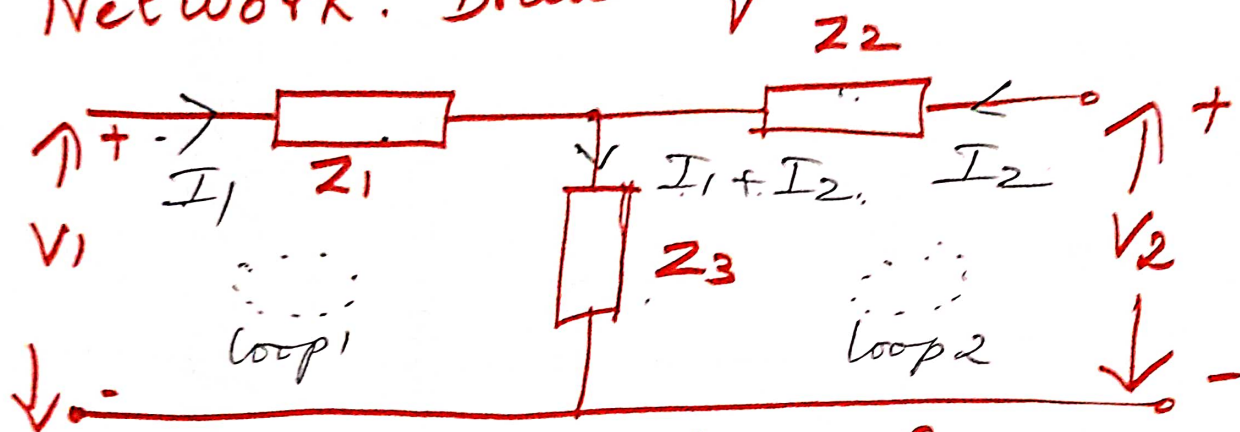
$$\Delta h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = (h_{11} h_{22}) - (h_{21} h_{12})$$

(11)

KVL $\rightarrow$ Z parameters.

KCL $\Rightarrow$ Y parameters.

Find Z parameters for the following Network. Draw equivalent circuit.



check the condition for symmetry and Reciprocity.

from positive of V_1 , I_1 flows thro' Z_1
from positive of V_2 , I_2 flows thro' Z_2
 $I_1 + I_2$ flows thro' Z_3 .

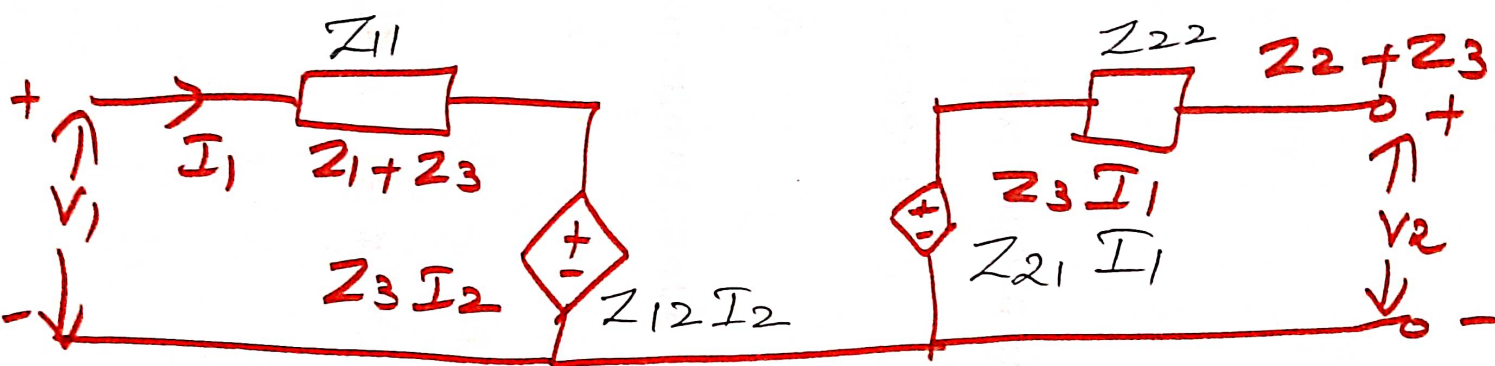
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} \Rightarrow$ sum of impedance in loop 1
 $= Z_1 + Z_3$

$Z_{22} = \text{Sum of impedance in loop 2}$ (12)
 $= Z_2 + Z_3$

$$Z_{12} = Z_{21} = Z_3$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



Not Symmetry

$$Z_{11} \neq Z_{22}$$

Reciprocity

$$Z_{12} = Z_{21}$$

$$Z_3 = Z_3$$

The circuit is reciprocal in nature.

The following readings are obtained experimentally for an unknown two port Networks. 13

	V_1	V_2	I_1	I_2
O/p open	100V	60V	10A	0
i/p open	30V	40V	0	3A

calculate Z parameters when i/p and o/p port are open

Open circuit condition = $I = 0$

Closed circuit condition = $V = 0$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{100}{10} = 10 \Omega$$

open circuited o/p port.
Take V_1 and I_1 when
o/p port is open circuited

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \frac{30}{3} = 10 \Omega$$

Open circuit i/p port.

Take V_1, I_2
when i/p port is
open circuited.

(14)

Take V_2, I_1
when o/p port
is open.

$$Z_{21} = \frac{V_2}{I_1} = \frac{60}{10} = 6\Omega$$

o/p port open. $I_2 = 0$

$$Z_{22} = \frac{V_2}{I_2} = \frac{40}{3} = 13.33\Omega$$

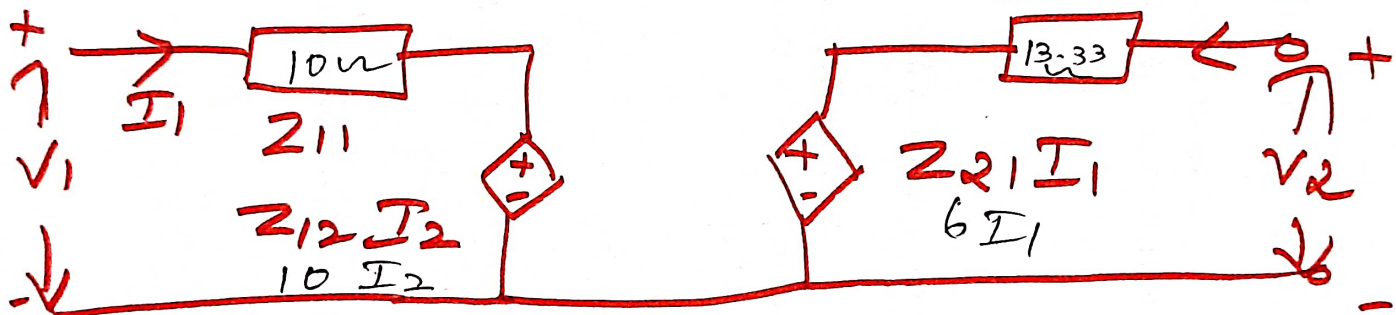
o/p port open. $I_1 = 0$

Symmetry

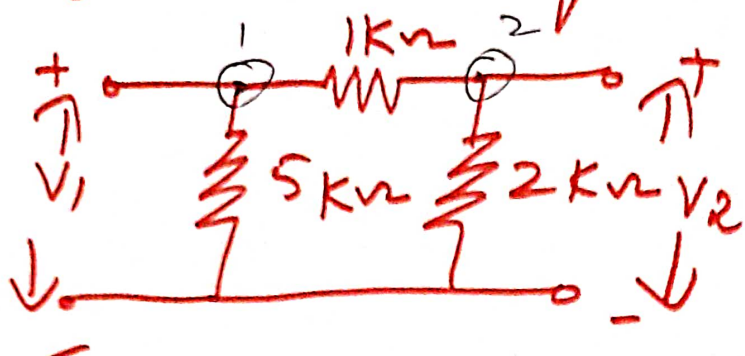
$$Z_{11} \neq Z_{22}$$

Reciprocity

$$Z_{12} \neq Z_{21}$$



An π attenuator is shown in fig. Find the 4 parameters and draw the equivalent 4 parameters 15



$\Rightarrow$ π
Circuit looks like π (pie)

For a π , it is possible to apply KCL.

$$\left| \frac{1}{R} \right| |V| = |I|$$

$$\left| \frac{1}{R} \right| = |Y|$$

$$= \begin{vmatrix} \left(\frac{1}{1} + \frac{1}{5} \right) \frac{1}{10^3} & \left(\frac{1}{1} \right) \frac{1}{10^3} \\ \left(\frac{1}{1} \right) \frac{1}{10^3} & \left(\frac{1}{2} + \frac{1}{1} \right) \frac{1}{10^3} \end{vmatrix}$$

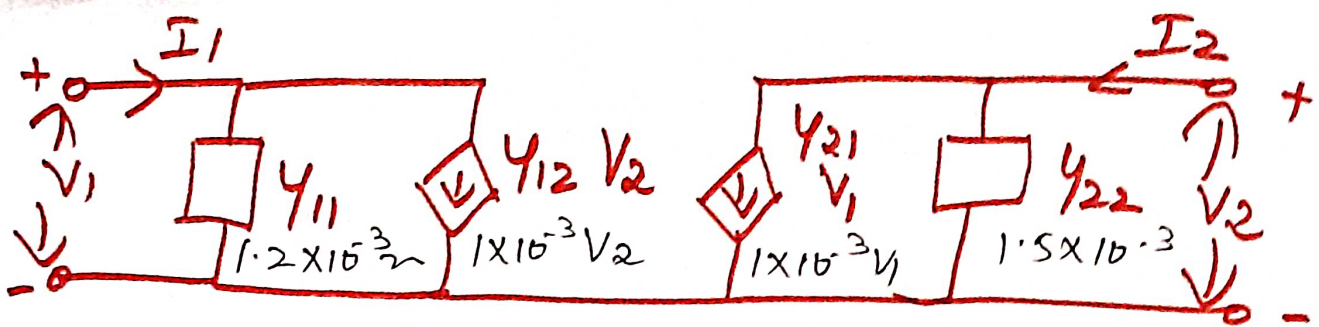
$$= \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} = \begin{vmatrix} 1.2 \times 10^{-3} & 1 \times 10^{-3} \\ 1 \times 10^{-3} & 1.5 \times 10^{-3} \end{vmatrix}$$

$$Y_{11} = 1.2 \times 10^{-3} \Omega^{-1}$$

$$Y_{12} = 1 \times 10^{-3} \Omega^{-1}$$

$$Y_{21} = 1 \times 10^{-3} \Omega^{-1}$$

$$Y_{22} = 1.5 \times 10^{-3} \Omega^{-1}$$



Symmetry

$$Y_{11} \neq Y_{21}$$

Reciprocity

$$Y_{12} = Y_{21}$$

The circuit is reciprocal in nature. On short circuit test, the currents and voltages were determined experimentally for an unknown two port network.

$$I_1 = 1 \text{ mA}$$

$$I_2 = -0.5 \text{ mA}$$

$$V_1 = 25 \text{ V}$$

$$V_2 = 0$$

Short circuit
o/p port

$$I_1 = 1 \text{ mA}$$

$$I_2 = -10 \text{ mA}$$

$$V_2 = 50 \text{ V}$$

$$V_1 = 0$$

Short circuit
i/p port.

Calculate Y parameters for short circuit conditions.

short circuit o/p port

17

$$V_2 = 0$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1 \times 10^{-3}}{25} = 40 \mu \Omega = 40 \times 10^{-6} \Omega$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-0.5 \times 10^{-3}}{25} = -20 \times 10^{-6} \Omega$$

short circuit i/p port

$$V_1 = 0$$

$$Y_{12} = \frac{I_1}{V_2} = \frac{1 \times 10^{-3}}{50} = -20 \times 10^{-6} \Omega$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{-10 \times 10^{-3}}{50} = -200 \times 10^{-6} \Omega$$

Symmetry

$$Y_{11} \neq Y_{21}$$

The circuit is

Reciprocity

$$Y_{12} = Y_{21}$$

Reciprocal.

